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**KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY**  
**UNIVERSITY EXAMINATION, 2016/2017 ACADEMIC YEAR**  
**THIRD YEAR, SECOND SEMESTER EXAMINATION**  
**FOR THE DEGREE OF BACHELOR OF SCIENCE**  
**(MATHEMATICS)**

Date: 8<sup>th</sup> August, 2016.  
Time: 11.00am – 1.00pm

**KMA 311 – PARTIAL DIFFERENTIAL EQUATIONS I**

**INSTRUCTIONS TO CANDIDATES**

**ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS**

**QUESTION ONE (30 MARKS)**

- a) State the definition of a partial differential equation. (2 Marks)
- b) Show that the direction cosines of the tangent at the point  $(x, y, z)$  to the conic  $ax^2 + by^2 + cz^2 = 1, x + y + z = 1$  are proportional to  $(by - cz, cz - ax, ax - by)$  (5 Marks)
- c) Find the integral curves of the sets of equations;

$$\frac{dx}{xz - y} = \frac{dy}{yz - x} = \frac{dz}{1 - z^2}$$

(6 Marks)

- d) Show that the orthogonal trajectories on the hyperboloid  $x^2 + y^2 - z^2 = 1$  of conics in which it is cut by the system of planes  $x + y = c$  are its curves of intersection with the surfaces  $(x - y)z = k$ , where  $k$  is a parameter. (7 Marks)
- e) Verify that the equation  $(yz + z^2)dx - xzdy + xydz = 0$  is integrable and find its primitive. (6 Marks)
- f) Eliminate the arbitrary function  $f$  from the equations  $x + y + z = f(x^2 + y^2 + z^2)$  (5 Marks)

**QUESTION TWO (20 MARKS)**

- a) Find the general solution of the partial differential equations.

$$x(x+y)p - y(x+y)q = (y-x)(2x+2y+z)$$

(8 Marks)

- b) Form partial differential equations by eliminating arbitrary constants from the following relations;  $z = (x-a)^2 + (y-b)^2$ ;

(4 Marks)

- c) Verify that the following equations are integrable and find their solutions.

$$(x^2z - y^3)dx + 3xy^2dy + x^3dz = 0$$

(8 Marks)

**QUESTION THREE (20 MARKS)**

- a) Find the equation of the integral surface of the differential equation

$$2y(z-3)p + (2x-z)q = y(2x-3)$$

which pass through the circles  $z=0, x^2 + y^2 = 2x$

(10 Marks)

- b) Find the surface which intersects with the surfaces of the system  $z(x+y) = c(3z+1)$

orthogonally and which passes through the circle  $x^2 + y^2 = 1, z = 1$ .

(10 Marks)

**QUESTION FOUR (20 MARKS)**

- a) Show that  $xp - yq = x, x^2p + q = xz$  are compatible and find their solution.

(10 Marks)

- b) Show that the Charpit's equations of the differential equation

$$(q^2 + 1)z^2 = 2pxz + x^2$$

have an integral  $qz = ax$  and find the corresponding complete integral of the equation.

(10 Marks)

**QUESTION FIVE (20 MARKS)**

Use Cauchy's method of characteristics to find the solution of the equation

$$z = \frac{1}{2}(p^2 + q^2) + (p-x)(q-y) \text{ through the point x-axis.}$$