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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2022/2023 ACADEMIC YEAR
FIRST YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF BUSINESS INFORMATION TECHNOLOGY

Date: 14th April, 2022
Time: 11.30am – 1.30pm

KMA 2208 - PROBABILITY AND STATISTICS 1

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Define a random variable, hence distinguish between the two types of random variables. (3 marks)

b) State two conditions that must be satisfied by a function $f(x)$ for it to be called a probability distribution of a continuous random variable X. (2 marks)

c) A discrete random variable X has a probability distribution given by

x	1	2	3	4	5
$P(X = x)$	0.1	0.25	0.4	0.2	0.05

Determine
i) Mean of X. (2 marks)
ii) Variance of X. (2 marks)

d) A random variable X has probability distribution given by

$$f(x) = \begin{cases} kx(1-x), & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$$

Find
i) Value of k. (2 marks)
ii) C.d.f. of X. (2 marks)
iii) Use the c.d.f to compute $P(\frac{1}{3} \le x \le \frac{1}{2})$. (2 marks)

e) A random variable X has a probability distribution given by $f(x) = \begin{cases} \frac{x}{6}, & x = 1,2,3 \\ 0, & elsewhere \end{cases}$

Determine the moment generating function, hence find the mean and variance of X. (6 marks)

f) If 3% of the electric bulbs manufactured by a company are defective. In a sample of 100 bulbs, let X be the number of defective bulbs found.

i) What is the probability distribution of X? (3 marks)

ii) Find the probability that at most three bulbs are defective. (3 marks)

g) Suppose that the number marks scored by students in a class is normally distributed with mean 45 and variance 225. What is the proportion of students who score between 40 and 60? (3 marks)

QUESTION TWO (20 MARKS)

a) A random variable X is uniformly distributed over the interval [a, b]. Show that

i) $E[X] = \frac{b+a}{2}$. (4 marks)

ii) $Var\ (X) = \frac{(b-a)^2}{12}$ (6 marks)

b) Prof Hinge travels always by plane. From past experience he feels that take off time is uniformly distributed between 80 and 120 minutes after check in. Let X be the time he waits for take-off after check in;

I) Write the distribution of X hence determine the probability that:

i) he waits for more than 15 minutes for take-off after check in. (3 marks)

ii) he waits between 5 to 10 minutes for take-off after check in. (3 marks)

II) Determine the mean and variance of X. (4 marks)

QUESTION THREE (20 MARKS)

a) Let X be a continuous random variable with pdf given by $f(x) = \begin{cases} \frac{1}{2}x, & 0 \le x \le 2 \\ 0, & otherwise \end{cases}$

Determine

i) $E(X)$. (3 marks)

ii) $Var(X)$. (3 marks)

iii) $E(4X^2 + 3X - 7)$. (2 marks)

iv) $Var(3X + 15)$. (2 marks)

b) The scores from a group of students in new test preparation score is assumed to be normally distributed with mean score of 1150 and a standard deviation of 150.

I) What is the proportion of students who;

i) Scored between 1000 and 1300? (3 marks)

ii) Scored above 1400? (3 marks)

II) Suppose that 65% of the students passed in the test, what is the minimum pass mark? (4 marks)

QUESTION FOUR (20 MARKS)

a) A random variable X has an exponential distribution given as $f(x) = \begin{cases} \frac{1}{\theta}\, e^{-\frac{x}{\theta}}, & x > 0 \\ 0, & otherwise \end{cases}$

I) Determine the mgf of X. (6 marks)

II) Use the mgf above to obtain.

i) $E(X)$. (3 marks)

ii) $Var(X)$. (4 marks)

b) Suppose that the amount of time one spends in a bank is exponentially distributed with mean 10 minutes.

i) What is the probability that a customer will spend more than 15 minutes in the bank? (3 marks)

ii) What is the probability that a customer will spend more than 15 minutes in the bank given that he is still in the bank after 10 minutes? (4 marks)

QUESTION FIVE (20 MARKS)

a) Distinguish between

i) Hypothesis and hypothesis testing. (2 marks)

ii) Point and Interval estimation. (3 marks)

iii) Null and alternative hypothesis. (2 marks)

iv) Type I and type II errors. (2 marks)

b) Suppose that the amount earned annually by police officers in the UK is normally distributed with mean $\mu = \$9000$ and variance $\sigma^2 = \$35{,}800$, both unknown. It is claimed that the amount earned by police officers in Brownsville city are way below what the entire UK police officer earn annually thus remuneration is necessary in the city. A random sample of 17 police officers in Brownsville has a mean annual income of $7,000. Can this claim be statistically justified? Use $\alpha = 1\%$. (6 marks)

Let the sample values be; 48, 70, 10, 20, 47, 50, 55, 30, 65, 42. Obtain 95% confidence interval of the population mean. (5 marks)