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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
FOURTH YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS)
KMA 411 - ACTUARIAL SCIENCE II

Date: 17th April, 2023
Time: 8:30 am-10:30am

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Explain what is meant by the "no arbitrage" assumption in financial mathematics. (2 Marks)

b) State the characteristics of fixed interest government bonds. (4 Marks)

c) A three-year forward contract is to be issued on a particular company share. The current market value of the share is Kshs 4.50 and a dividend of Kshs 0.20 per share has just been paid. The parties to the contract assume that the future quarterly dividends will increase by 1% per quarter-year compound for the first two years and by 1½% per quarter-year compound for the final year. Assuming a risk-free force of interest of 5% per annum, and no arbitrage, calculate the forward price. (6 Marks)

d) Explain reasons why interest rates vary over time. (6 Marks)

e) A lump sum of Kshs14,000 will be invested at time 0 for 4 years at a constant annual rate of interest i. $(1+i)$ has a log-normal distribution with mean 1.05 and variance 0.007. What is the probability that the investment will accumulate to more than Kshs20,000 in 4 years' time? (6 Marks)

f) Explain the three theories that explain the term structure of interest rates. (6 Marks)

QUESTION TWO (20 MARKS)

A fixed interest stock is redeemable at 106% in 15 years' time and pays interest at 9% *pa* payable half-yearly in arrears.

a) Calculate the price an investor should pay to obtain a gross redemption yield of 9% *pa*. (2 Marks)

b) Instead of purchasing the stock, the investor decides to agree a forward contract to buy the security in six years' time, immediately after the coupon payment then due. Calculate the forward price based on a risk-free rate of return of 6% *pa* effective and no arbitrage. The current price of the stock is that calculated in part (i). (3 Marks)

c) Three years later, the price of the security is such that the gross redemption yield is still 9%. Calculate the value of the forward contract if the risk-free yield has not changed. (6 Marks)

d) Calculate the yield obtained if the investor sold the forward contract after three years. (2 Marks)

QUESTION THREE (20 MARKS)

A fixed interest security of nominal amount Kshs1,000,000 is to be issued paying coupons quarterly in arrears at a rate of 6% per annum. The security is to be redeemed with a capital payment of Kshs105 per Kshs100 nominal on a coupon date between 20 and 25 years after the date of issue, inclusive. The date of redemption is at the option of the borrower. An investor, who is liable to income tax at 20% and capital gains tax of 25%, wishes to purchase the entire security at the date of issue, at a price that ensures she achieves a net effective yield of at least 4.9% per annum.

a) Determine whether the investor would make a capital gain if she holds the security until redemption. (5 Marks)

b) Explain how your answer to part (i) influences the assumptions made in calculating the price, the investor should pay. (3 Marks)

c) Calculate the maximum price that the investor should pay per Kshs100 nominal. (8 Marks)

d) Explain, without carrying out any further calculations, how your answer to part (iii) would change if the coupons had been payable half-yearly in arrears. (4 Marks)

QUESTION FOUR (20 MARKS)

a) The annual effective forward rate applicable over the period from t to $t + r$ is defined as $f_{t,r}$ where t and r are measured in years.

You are informed that $f_{0,1} = 4\%$, $f_{1,1} = 5\%$, $f_{2,1} = 6\%$ and $f_{3,1} = 7\%$.

i) Determine the gross redemption yield at issue for a four-year bond, redeemable at par, with a 4% coupon payable annually in arrears. (7 Marks)

ii) Explain why the gross redemption yield in part (ii) is lower than $f_{3,1}$. (3 Marks)

b) An individual buys an annuity from an insurance company for a single lump sum premium. The annuity will pay Kshs10,000 annually in arrears for 15 years. The insurance company invests the premium in a fixed-interest bond which pays coupons at the rate of 6% per annum annually in arrears and is redeemable at par in exactly nine years.

i) Calculate the duration of the annuity at an interest rate of 5% per annum effective (3 Marks)

ii) Calculate the duration of the bond at an interest rate of 5% per annum effective. (3 Marks)

iii) Explain whether the insurance company will make a profit or a loss if interest rates decrease slightly at all terms. (4 Marks)

QUESTION FIVE (20 MARKS)

a) Describe the main features of Eurobonds.

(4 Marks)

b) On 1 February 2017, an investor was considering purchasing ordinary shares in Actuarial PLC. Dividends are payable annually, and a dividend of Kshs0.40 per share had just been paid. At the date of purchase, dividends were expected to grow each year on a compound basis. The rate of growth was expected to be 5% in the first year, 4% in the second year and 3% per annum thereafter.
The investor was not entitled to the dividend which had just been paid.

i) Calculate the maximum price per share the investor would have been prepared to pay at this date to give a rate of return of 9% per annum effective, assuming the investor holds the share in perpetuity.

(8 Marks)

The investor purchased a holding of shares on 1 February 2017 at a price of Kshs7.00 per share and sold the holding at a price of Kshs7.50 per share on 1 February 2019, immediately after receiving the dividend payment then due.

ii) Calculate the effective annual real rate of return achieved by the investor between 1 February 2017 and 1 February 2019 using the following information:

Date	*Inflation index*	*Dividend per share*
1 February 2017	211.0	Kshs0.400
1 February 2018	215.7	Kshs0.428
1 February 2019	221.2	Kshs0.449

(8 Marks)