KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2022/2023 ACADEMIC YEAR
FIRST YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF BUSINESS INFORMATION TECHNOLOGY

Date: 25th July, 2022

Time: 11.30am −1.30pm

KMA 2102 - CALCULUS FOR BUSINESS INFORMATION

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a)   Find the rate of change of the following functions over the given intervals

$f(x) = x^2 + 1$ $[-1, 1]$ (3 marks)

b)   Find the slope of the curve $y = x^2 - 3$ at the point $P(2, 1)$ (2 marks)

Hence find the equation of the tangent at P.   (3 marks)

c)   Evaluate $\lim\limits_{x \to 1} \dfrac{x^2 + x - 2}{x^2 - x}$ (3 marks)

d)   Find the derivative of $y = x^3 + \dfrac{4}{3}x^2 - 5x + 1$ (2 marks)

e)   If $f(x) = x^2$ and $g(x) = x + 2$, find

i) $(f \circ g)(x)$ (3 marks)

ii) $g(f(2))$ (2 marks)

f)   Given $f(x) = x^2 + 2$, find $\dfrac{df}{dx}$ using first principle method.   (4 marks)

g)   Use the product rule to find $y = (x^2 + 1)(x^3 + 3)$ (3 marks)

h)   Show that the derivative of $y = \tan x$ is $\sec^2 x$.   (5 marks)

QUESTION TWO (20 MARKS)

a)   Find the derivatives of the following functions

    i) $y = \dfrac{\sin x}{x}$ (3 marks)

    ii) $y = \sin x \cos x$ (3 marks)

    iii) $y = 5e^x + \cos x$ (2 marks)

b)   Differentiate $y = \dfrac{x^2 + e^x}{x}$ using the product rule   (3 marks)

c)   Use the chain rule to find the derivative of $y = \sin\ (1 - 5x)$ (4 marks)

d)   Find $f'(x)$ for the function $f(x) = \dfrac{8}{7x}$ (5 marks)

QUESTION THREE (20 MARKS)

a)   Determine the coordinates of the stationary points on the curve whose equation is
$y = 2x^3 - 21x^2 - 60x + 12$ and determine their nature.

                                                                                     (5 marks)

b)   Find the domain and range of $f(x) = 2 + \sqrt{x - 1}$ (5 marks)

c)   A shot is fired from the top of a tower which is at a height $h(t) = 100 + 2t - 2t^2$ after t seconds.
Find:

    i)   Its velocity after 2 second   (2 marks)

    ii)   Its maximum height   (2 marks)

    iii)   Its velocity as the bullet hits the ground   (2 marks)

d)   Use implicit differentiation to find an equation of the line tangent to the equation $x^2 + y^2 = 4$ at
the point $(1,2)$.   (4 marks)

QUESTION FOUR (20 MARKS)

a)   A circular sheet of material has a radius of 4cm. Find the rate at which the area is increasing with
respect to the radius hence find the approximate increase in the area if the radius is increased to 4.2
cm.   (5 marks)

b)   The distance covered by a moving body after t seconds is given as $s(t) = t^3 - 3t^2 + 2$. Find:

    i)   Acceleration of the body at $t = 2$.   (3 marks)

    ii)   Time the acceleration will be zero.   (2 marks)

c)   By differentiating implicitly find $\frac{d^2y}{dx^2}$ if $x^2 = 1 + y^2$, leave your answer in terms of x and y.

                                                                                   (5marks)

d)   A spherical balloon is inflated at the rate of $10cm^3$ per minute. How fast is the radius of the
balloon increasing when the radius is 5cm?   (5 marks)

QUESTION FIVE (20 MARKS)

a)   A ball thrown upwards from a building attains a height of $h(t) = 800 + 400t - 16t^2$ m after t
        seconds. Find:

      i)   The time it attains its maximum height   (3 marks)

      ii)   The maximum height   (3 marks)

      iii)   The velocity after 15 seconds.   (3 marks)

      iv)   Sketch the curve between $t = 0$ to $t = 20$ seconds   (2 marks)

b)   Sketch the graph of $y = \frac{1}{3}x^3 + x^2 - 8x + 1$.   (4 marks)

c)   Evaluate the limits

      i) $\lim_{x \to -3} \frac{x^2 - 9}{x + 3}$ (3 marks)

      ii) $\lim_{x \to \infty} \frac{x^4 + x}{6x^3 - x^2}$ (2 marks)