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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
SECOND YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS
KMA 2204: LINEAR ALGEBRA II

Date: DECEMBER 2024
Time:

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

a) Find the angle between (1,0) and (1,1) in $\mathbb{R}^2$ where the inner product is defined as $\langle x, y\rangle = 2x_1y_1 + x_1y_2 + x_2y_1 + x_2y_2$. **(3 Marks)**

b) Find the coordinate vector of $v = (3,1)$ relative to the basis $B = \{(1,1),(-1,1)\}$. **(3 Marks)**

c) Consider the bases $B = \{(1,-3),(-2,4)\}$ and $C = \{(7,-9),(-5,7)\}$ for $\mathbb{R}^2$.

i. Find the Transition matrix from B to C. **(3 Marks)**

ii. Find the transition matrix from C to B. **(3 Marks)**

d) Suppose $C[0,1]$ is the vector space for continuous real-valued functions with an inner product space defined by $< f(x), g(x) >= \int_{-1}^{1} f(x)g(x)dx$. Verify the **Cauchy-Schwarz inequality** for $f(x)$=1 and $g(x)$=x. **(3 Marks)**

e) Given the inner product defined by $< p(x), q(x) >= a_0b_0 + a_1b_1 + a_2b_2$, where $p(x) = a_0+a_1,x + a_2x^2, q(x) = b_0 + b_1x + b_2x^2$, let $p(x) = 1 - 2x^2$ and $q(x) = 4 - 2x + x^2$, compute:

i. $\langle p, q\rangle$ **(1 Mark)**

ii. $||q||$ **(2 Marks)**

iii. $d(p,q)$ **(2 Marks)**

f) Let $A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$. Obtain the eigenvalues of matrix A. **(4 Marks)**

g) Write down the quadratic matrix associated with the following quadratic form

$$3x^2 - 8xy - 5y^2 + 6xz - 4yz - 4z^2 = 48.$$ **(2 Marks)**

h) If $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$, $u = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$, and $v = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$. Determine if u and v are eigenvectors of A?

(4 Marks)

QUESTION TWO: (20MARKS)

a) Find the orthogonal basis for the function space $\{1, 2t - 1, 12t^2\}$ where the inner product is defined as $< p(t), q(t) >= \int_0^1 p(t)q(t)dt$. **(7 Marks)**

b) Determine if the given matrix A is diagonalizable. Hence find a matrix P which diagonalizes

$$A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}.$$

(8 Marks)

c) Given the matrix $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Write matrix A in the form PDP^{-1}, where D is diagonal and hence find A^3. **(5 Marks)**

QUESTION THREE: (20MARKS)

a) Convert the set $S = \{(1, -1, 1), (1, 0, 1), (1, 1, 2)\}$ into an orthonormal basis for $\mathbb{R}^3$. **(7 Marks)**

b) Show that the following set $\left\{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(\frac{-\sqrt{2}}{6}, \frac{\sqrt{2}}{6}, \frac{2\sqrt{2}}{3}\right), \left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)\right\}$ is an orthonormal basis.

(5 Marks)

c) The product of two eigenvalues of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third eigenvalue of matrix A. **(4 Marks)**

d) Find a, b so that $\begin{bmatrix} a \\ b \\ 2 \end{bmatrix}$ is orthogonal to $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$. **(4 marks)**

QUESTION FOUR: (20MARKS)

a) i. Show that the matrix A= $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ is nonsingular. **(2 Marks)**

ii. Find the QR factorization of matrix A. **(6 Marks)**

b) Consider the bases $B = \{u_1, u_2\} = \{(1, 0), (0, 1)\}$ and
$B' = \{v_1, v_2\} = \{(3, 1), (2, -1)\}$ for $\mathbb{R}^2$.

i. Find the Transition matrix from B to C. **(3 Marks)**

ii. Find the transition matrix from C to B. **(3 Marks)**

iii. Compute the coordinate matrix $[v]_{B'}$, where $v = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$. **(2 Marks)**

c) Give that u and v are orthogonal vectors, prove that $\|u + v\|^2 = \|u\|^2 + \|v\|^2$. **(4 Marks)**

QUESTION FIVE: (20MARKS)

a) Identify the curve which is represented by the following quadratic equation by first putting it into standard conic form $x^2 - 8xy - 5y^2 = 0$. **(8 Marks)**

b) Write down the quadratic matrix associated with the following quadratic form
$5x^2 - 3y^2 - xz + 8yz - 2z^2 = 0$. **(3Marks)**

c) Given the matrix A= $\begin{bmatrix} 2 & 4 & 3 \\ -1 & -6 & -3 \\ -1 & 3 & -2 \end{bmatrix}$. Determine if the given vectors $\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$ is an eigenvectors of matrix A and hence, find the eigenvalue of A associated to this eigenvector. **(4 Marks)**

d) i) State the Cayley-Hamilton theorem **(1 Mark)**

ii) Use Cayley-Hamilton theorem to find A^{-1} (if it exists) for the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & -1 & 6 \\ 0 & 8 & 1 \end{bmatrix}.$$ **(4 Marks)**