



Kasarani Campus
Off Thika Road
P. O. Box 49274, 00101
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMEN'S UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2024/2025 ACADEMIC YEAR
FIRST YEAR, SEMESTER TWO EXAMINATION
BACHELOR OF EDUCATION (ARTS)
KMA 2201: INTEGRAL CALCULUS

Date: 13th December 2024

Time: 8.30am-10.30am

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Evaluate the following integrals:

i) $\int x^2 \sqrt{x^3 + 5} dx$ (4 marks)

ii) $\int_1^4 (5x - 2\sqrt{x} + \frac{32}{x^3}) dx$ (4 marks)

iii) $\int \frac{x dx}{(x-1)(x+1)^2}$ (3 marks)

b) The parabola $y = 2 - x^2$ and the line $y = -x$ enclose an area. Use integration methods to find the size of the area enclosed. (5 marks)

c) The region bounded by the graph

$$x^2 = y - 2, 2y - x^2 - 2 = 0, x = 0, x = 1 \text{ is rotated } 360^\circ \text{ about the } x\text{-axis.}$$

Find the volume of the resulting solid (4 marks)

d) A stone falls freely from the top of a cliff and its velocity v , after t seconds is given by $10t \text{ m/s}$. Find the distance S if it falls in 5 seconds (5 marks)

e) Obtain the reduction formulae for $I_n = \int x^n e^x dx$.

Hence evaluate $\int x^4 e^x dx$ (5 marks)

QUESTION TWO (20 MARKS)

a) Use Simpson's rule with 9 ordinates correct to 4 decimal places to estimate (7 marks)

$$\int_2^4 \frac{5 \ln 2x}{5 + \ln 2x} dx$$

b) Show that the improper integral $\int_1^\infty \frac{1}{x^2} dx$ is convergent (4 marks)

c) Evaluate $\int x^3 \sqrt{1 - x^2} dx$ using Integration by parts. (4 marks)

d) Obtain the reduction formulae for $I_n = \int \sin^n x dx$.

Hence evaluate $\int \sin^4 x dx$ (5 marks)

QUESTION THREE (20 MARKS)

- a) Find the integral $\int \sin^3 x \, dx$ (5 marks)
- b) Find $\int \frac{1+x^2}{\sqrt[4]{3x+x^3}} \, dx$ (6 marks)
- c) Find the area enclosed by the curves $y=2x-x^2$ and $y = 2x^2 - 4x$ (5marks)
- d) Verify the fundamental theorem of calculus using the equation
 $2x^3 - 3x^2 + x - 2$ on $[2, 5]$. (4 marks)

QUESTION FOUR (20 MARKS)

- a) Find $\int \frac{dx}{\sqrt{1+x^2}} \, dx$ (5 marks)
- b) Use partial fractions to evaluate
 $\int \frac{2x^2-11x+5}{(x^2+2x-5)(x-3)} \, dx$ (6 marks)
- c) i) Evaluate $\int_0^4 \frac{1}{\sqrt{1+x^3}} \, dx$ using **Simpsons Rule** with $n = 3$ (5 marks)
ii) Estimate the error involved in this approximation (4 marks)

QUESTION FIVE (20 MARKS)

- a) Find $\int 2x^2 \sec(4\pi x^3 + 6) \, dx$ (5 marks)
- b) Evaluate the integral $\mathbf{I} = \int_1^2 \frac{1}{x} \, dx$ using trapezoidal rule using 4 equal sub intervals . (7 marks)
- c) Evaluate the following definite integrals
 $\int_0^\pi (x + 1) \sin x \, dx$ (4 marks)
- d) Using for integration find
 $\int \cos 5x \cos 3x \, dx$ (4 marks)