Kasarani Campus
Off Thika Road
Tel. 2042692 / 3
P. O. Box 49274, 00100
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
FOURTH YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS
KMA 307- MATHEMATICS FOR MODELING

Date: 19TH APRIL 2023
Time: 8:30 AM-10:30AM

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

a) Define the following terms:
   i) Mathematical Modeling (1 Mark)
   ii) Empirical model (1 Mark)
   iii) Deterministic model (1 Mark)
   iv) Stochastic model (1 Mark)

b) Solve the difference equation below;
   $\boldsymbol{x_t = 2x_{t-1} + 2x_{t-2},\ \ x_0 = 0\ \ x_1 = 1}$ Hence find x_{20} and x_{50} (4 Marks)

c) In marketing a certain item, a business has found that the demand for the item is represented by the function $\boldsymbol{P = \frac{55}{\sqrt{x}}}$ and the cost of producing $\boldsymbol{x}$ items is given by $\boldsymbol{C = 0.4x + 700}$
   Find the price per item that gives the maximum profit. (4 Marks)

d) Consider the following linear systems,

$$\boldsymbol{X' = 3x - 1.4xy \quad and}$$
$$\boldsymbol{Y' = -y + 0.8xy}$$

   i) What is the x and y nullclines. (2 Marks)
   ii) Determine the equilibrium points. (2 Marks)

e) A certain Radioactive material is known to decay at a rate proportional to the amount present. Initially, there was 300kgs of the material. After 2 years, 5 percent had Decayed.
   Find:
   i) An expression for any mass present at any time t (1 Mark)
   ii) A time required for 10 percent of the original amount to decay. (2 Marks)
   iii) The amount present after 8 years. (1 Mark)
   iv) The Half-life of the material. (2 Marks)

f) Gold and silver coins are allocated among three Urns labelled I, II and III as in the table below.

Urn	Gold	Silver
I	4	8
II	3	9
III	6	6

   An Urn is selected at random and a coin is selected from that Urn. What is the probability of selecting a gold coin. (4 Marks)

g) A Zinc Plate is heated to a temperature of 100^0C. Then at time $t = 0$ it is placed in water which is maintained at a temperature of 16^0C. At the end of 3 minutes the temperature of the ball is reduced to 90^0 Find the time at which the temperature of the ball is reduced to 31^0 C. (4 Marks)

QUESTION TWO (20 MARKS)

a) Differentiate between a stable equilibrium and an unstable equilibrium. (2 Marks)

b) Consider the following non-linear systems, $X' = x(1 - x - 2y)$ *and*

$$Y' = y(1 - 2x - y)$$

Find all the equilibrium points and determine their stability. Justify your answer. (10 Marks)

c) As the salt (potassium nitrate) dissolves in methanol, the number X(t) of grams of salt in solution after t seconds satisfies the differential equation $\frac{dy}{dt} = 0.8x - 0.004x^2$.

i) What is the maximum amount of the salt that will ever dissolve in the methanol? (3 Marks)

ii) ***If x=50 when t=0***, how long will it take for an additional 50g of salt to dissolve? (5 Marks)

QUESTION THREE (20 MARKS)

a) An investor puts Kshs. 400,000 in a financial fixed savings account which pays 8 percent compound interest p.a. Assuming no injection or withdrawals, find the amount after 5 years if the amount is compounded;

i) Yearly (2 Marks)

ii) Quarterly (2 Marks)

iii) Monthly (2 Marks)

iv) Continuously (2 Marks)

v) How long will it take the amount to double if the interest is compounded

i) Continuously (2 Marks)

ii) Twice a year (2 Marks)

b) There are two people, Bill and George who goes target shooting. Both shoot at the target at the same time. Suppose the probability of Bill hitting the target is 0.7 while George's is 0.4.

i) Given that the target was hit exactly once, what is the probability that it was Georges hit. (5 Marks)

ii) Given that the target is hit, what is the probability that George hit it? (3 Marks)

QUESTION FOUR (20 MARKS)

a) The population of Thika town center at any time t given by $N(t)$ is assumed to satisfy the logistic growth law; $\frac{dM}{dt} = \frac{1}{300} M(5,000 - M)$ ***find M(t)*** (7 Marks)

b) A man runs the 100 meters race as follows. He takes off and accelerates for x seconds such that his speed V(t), at time t is given by, $0 \le t \le x$.

Thereafter, he runs at a constant speed reached at $t = x$

i) Sketch the speed-time graph. (2 Marks)

ii) If he completes the race in 10 seconds, determine the value of x (to 2 decimal places). (6 Marks)

iii) Determine also the maximum speed reached (to 2 decimal places) and the distance covered during acceleration to the nearest meter. (5 Marks)

QUESTION FIVE (20 MARKS)

a) Consider a prolific breed of Rabbits whose Birth and Death rates β *and* δ, are each proportional to the rabbit's population $p = p(t)$ **with** $\beta > \delta$.

i) Show that $(t) = \frac{P_o}{1-kP_o t}$, for some constant K.

Note that $P(t) \to \infty$ *as* $t \to \frac{1}{kP_o}$. The doomsday situation. (6 Marks)

ii) Suppose that $P_o = 6$ and that there are 9 rabbits after 10 months, when does doomsday occur? (4 Marks)

b) The selling price x in Kshs of each item is related to the total sales, S by $S = 500 - x^2$. The cost of production, C is related to the total sales by $C = 100 + \frac{S}{5}$. Determine the selling price, to one decimal place, that will result in the largest possible profit. What is the largest possible profit?(6 Marks)

c) Derive the solution of the following difference equation: (4 Marks)

$x_t = 6x_{t-1} - 9x_{t-2},\ x_0 = 4\ x_1 = 6$ ***Hence find*** x_{20} ***and*** x_{50}