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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
END OF SEMESTER EXAMINATIONS
FOR THE DEGREE OF BACHELOR OF MATHEMATICS

KMA 207: THEORY OF ESTIMATION

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE COMPULSORY (30 MARKS)

a) Distinguish between
   i) Consistency and Efficiency. (2 marks)
   ii) Point and Interval estimator. (2 marks)

b) Consider a random variable X with pdf

$$f(x,\theta) = \begin{cases} \frac{1}{\theta}\, e^{-\frac{x}{\theta}}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

   i) Find the moment estimator of θ. (4 marks)
   ii) Show that the moment estimator obtained in i) is unbiased. (2 marks)

c) Let $x_1, x_2 \dots, x_n$ be a random sample of size n from be a random variable X with p.m.f

$$f(x,p) = \begin{cases} p^x\,(1-p)^{1-x}, & x = 0, 1 \\ 0, & \text{Otherwise} \end{cases}$$

Determine the sufficient statistic for p. (5 marks)

d) Let $x_1, x_2 \dots, x_n$ be a random sample of size n from be a random variable X with a normal density

$$f(x,\mu) = \begin{cases} \frac{1}{2\sqrt{\pi}}\, e^{-\frac{1}{4}(x-\mu)^2}, & -\infty < x < \infty \\ 0, & \text{Otherwise} \end{cases}$$

Find the maximum likelihood estimator of μ. (5 marks)

e) Consider a random sample $y_1, y_2, y_3, \dots, y_n$ of size n from Y with pdf

$$f(y,\alpha) = \begin{cases} \alpha\, e^{-\alpha y}, & y > 0 \\ 0, & \text{Otherwise} \end{cases}$$

Find the Cramer-Rao lower bound of $\psi\,(\alpha) = \frac{1}{\alpha}$.

(5 marks)

f) To compare customer satisfaction levels of two competing cable television companies, 174 customers of Company 1 and 355 customers of Company 2 were randomly selected and were asked to rate their cable companies on a five-point scale, with 1 being least satisfied and 5 most satisfied. The survey results are summarized in the following table below.

Company 1	Company II
$n_1 = 174$	$n_2 = 355$
$\bar{x}_1 = 3.51$	$\bar{x}_2 = 3.24$
$s_1 = 0.51$	$s_2 = 0.52$

Construct a point estimate and a 99% confidence interval for $\mu_1 - \mu_2$, the difference in average satisfaction levels of customers of the two companies as measured on this five-point scale.

(5 marks)

QUESTION TWO (20 MARKS)

a) State the three Cramer-Rao regular conditions. (3 marks)

b) Show that under the regular conditions above, the Cramer-Rao inequality is given by

$$\mathrm{Var}(T) \le \frac{(\psi\,'\,(\theta))^2}{I(\theta)}$$

Where $I(\theta) = E\left(\frac{\partial \log L}{\partial \theta}\right)^2 = -E\left(\frac{\partial^2 \log L}{\partial^2 \theta}\right)$, $\psi\,(\theta)$is any function of θ and T is unbiased estimator of $\psi\,(\theta)$. (11 marks)

c) Let $x_1, x_2 \ldots, x_n$ be a random sample of size n from be a random variable X with a normal density

$$f(x,\mu) = \begin{cases} \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, & -\infty < x < \infty \\ 0, & \text{Otherwise} \end{cases}$$

i) Find the Cramer-Rao lower bound of $\psi(\mu) = \mu$. (4 marks)

ii) Hence find the UMVUE of μ, if it exists. (2 marks)

QUESTION THREE (20 MARKS)

a) Let $x_1, x_2, \ldots, x_m$ be a random sample of size m from $X \sim N(\mu_1, \sigma_1^2)$, where both parameters are unknown. Let $y_1, y_2, \ldots, y_n)$ be another independent random sample from $Y \sim N(\mu_2, \sigma_2^2)$, also both parameters are unknown. Derive $100(1-\alpha)\%$ confidence intervals for the difference in population means $\mu_1 - \mu_2$, where both X and Y are independent variables. (10 marks)

b) Two independent random samples were obtained from two independent random variables $X \sim N(\mu, \sigma^2)$ and $Y \sim N(\mu_2, \sigma^2)$, that is $\sigma_1^2 = \sigma_2^2 = \sigma^2$ but unknown. The observations are as follows;

X: 20, 33, 57, 22, 44, 31, 33, 40

Y: 44, 55, 36, 65, 38, 45, 54, 50, 48, 62

Obtain 99% confidence intervals for the difference in the two population means. (10 marks)

QUESTION FOUR (20 MARKS)

a) A random sample of size 10 had a mean $\bar{X} = 20$ and a standard deviation $s = 18$. Obtain 95% confidence intervals for true population variance σ^2. (5 marks)

b) A random variable X has a pdf given by the gamma density

$$f(x; \alpha, \beta) = \begin{cases} \frac{1}{\beta^{\alpha}\,\Gamma\alpha}\, x^{\alpha-1}\, e^{-\frac{x}{\beta}}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

i) Find the moment estimators of α and β. (10 marks)

ii) The failure time in years of a certain machine observed over time are;

0.41, 0.58, 0.75, 0.85, 1.00, 1.08, 1.17, 1.25, 1.35

If this failure time can be model using a gamma distribution above, determine the moment estimates of α *and* β. (5 marks)

QUESTION FIVE (20 MARKS)

a) Let $\underline{X} = (x_1, x_2 \dots , x_n)$ be a random sample of size n from be a random variable X with p.m.f

$$f(x, p) = \begin{cases} p^x(1-p)^{1-x}, & x = 0, 1 \\ 0, & \text{Otherwise} \end{cases}$$

Find

i) The joint probability distribution $P(\underline{X},\ T)$. (4 marks)

ii) The distribution of the statistic T. (4 marks)

iii) The conditional probability distribution $P(\underline{X}/T)$, hence show that $T = \sum x_i$ is sufficient for p. (3 marks)

b) A response variable Y is related with two variables X_1 and X_2 in the form $Y = a_0 + a_2 X_1 + e_i$. Data on seven sampled items are as shown in the table below

Y	12	22	17	15	21	23	25
X_1	5	8	7	6	8	9	11

i) Use matrix notation to fit the given linear model. (7 marks)

ii) Estimate the variance of each parameter given that $e_i \sim N(0, 1)$. (2 marks)