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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2022/2023 ACADEMIC YEAR

SECOND YEAR, FIRST SEMESTER EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION (ARTS)

Date: 1st August, 2022

Time: 11.30am –1.30pm

KMA 2204 - LINEAR ALGEBRA 11

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) i) Compute $< u, v >$ using the inner product $\mathbb{R}^2$ defined by $< u, v >=4u_1v_1 + 2u_2v_2$ where $u = (-1,3), v = (4,5)$. (2 Marks)

ii) Use the inner product defined by $< f, g >=\int_0^1 f(x)g(x)dx$ to compute the $< f, g >$ for the vectors of $f(x) = x$ and $g(x) = x^2 + 1$ in $C[0,1]$. (3 Marks)

b) i) Verify the Cauchy-Schwartz inequality for the vectors $u = (-3, 3), v = (5, 6)$ $in\ R^2$

(3 marks)

ii) Find the norm of the vector $q(x) = 1 + x$ using the integral inner product defined as

$$< p(x), q(x) >= \int_{-1}^{1} p(x)q(x)dx$$

(3 marks)

c) Find the matrix A of the quadratic form $Q(x) = 3x^2 + 2y^2 + z^2 + 4xy + 4yz.$

What are the eigenvalues of A? Is A positive definite? (5marks)

d) For a matrix $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$

i) Write down the characteristic equation (2 marks)

ii) Find the eigenvalues. (2 marks)

iii) Find the eigenvectors corresponding to each eigenvalue (3marks)

e) Find the angle between the vectors $x = (1,4,-1) and\ y = (-1\ 4,2)$ using the Euclidean inner product.

(3 marks)

f) Given that $B = \{p_1, p_2, p_3\} = \{-2, -4x\ , 5x^2 - 1\}$ is a basis for P_2.
Find the coordinate vector of $p = 4 - 2x + 3x^2$ with respect to P. (2 marks)

QUESTION TWO (20 MARKS)

a) Consider the standard basis for $p_2, B = \{1, 1 + x, 1 + x + x^2\}, and\ C = \{2 + x + x^2\ , x + x^2, x\}$.

i) Find the transition matrix from B to C. (6 marks)

ii) Use the result in (i) to find the change of basis matrix from C to B.

(8 marks)

b) Express the matrix $A = \begin{bmatrix} -1 & -4 \\ -4 & 5 \end{bmatrix}$ in the form PDP^{-1}, where D is diagonal.

(6 marks)

QUESTION THREE (20 MARKS)

a) Find a matrix P which orthogonally diagonalizes

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

(10 marks)

b) Identify the curve, by first using a suitable rotation to put the following quadratic equation into standard conic section.
$5x^2 - 4xy + 5y^2 - 48 = 0$

(10 marks)

QUESTION FOUR (20 MARKS)

a) Let

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

i) Find eigenvalues of A.

(6 marks)

ii) Use Cayley-Hamilton theorem to find A^3 and A^{-1} (if it exists) for the matrix A.

(8 marks)

b) Verify that the following is an inner product on P_2.

$< p(x), q(x) >= a_0b_0 + a_1b_1 + a_2b_2, where$

$p(x) = a_0 + a_1, x + a_2x^2, q(x) = b_0 + b_1x + b_2x^2$ (6 marks)

QUESTION FIVE (20MARKS)

a) Let $\mathbb{R}^3$ have the Euclidean inner product. For which values of k are u and v orthogonal? Given $= (k, k, 1),\ v = (k, 5, 6)$. (6 Marks)

b) Tranform the set $S = \{1, x, x^2\}$ which is a basis of P_2 into an orthonormal basis with respect to the integral inner product defined as $< p, q >= \int_{-1}^{1} p(x)q(x)dx$.

(14 Marks)