Kasarani Campus
Off Thika Road
P. O. Box 49274, 00101
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
FIRST YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS
(SPECIAL EXAMINATION)

KMA 209 ALGEBRA

Date: 12TH AUGUST, 2024
Time: 8:30 AM – 10:30 AM

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

a) Define the following terms
   (i) Group
   (ii) Binary operation
   (iii) Permutation **(6 Marks)**

b) Prove that for all $a,b \in G$ then $(ab)^{-1} = b^{-1}a^{-1}$ **(4 Marks)**

c) Define $*$ on Q^+ by $a*b = \frac{ab}{2}$. Show that $(Q^+,*)$ is a group. **(4 Marks)**

d) Define an abelian group and prove that every cyclic subgroup is abelian. **(4 Marks)**

e) Show that every division ring is a ring without zero divisor. **(5 marks)**

f) Define transposition and list the even and odd permutations in S_3 **(4 Marks)**

g) Prove that every field is an integral domain. **(3 Marks)**

QUESTION TWO: (20 MARKS)

a) Let G denote the set of all ordered pairs of real numbers with non-zero first component of the binary operation $*$ is defined by $(a,b)*(c,d) = (ac, bc+d)$. Show that $(G,*)$ is a non-abelian group. **(8 Marks)**

b) Let A be a non-empty set and let S_A be the collection of all permutations of A. Show that S_A is a group under permutation multiplication. **(7 Marks)**

c) An identity element (if it exist) of mathematical system $(S,*)$ is unique. Prove. **(5 Marks)**

QUESTION THREE: (20 MARKS)

a) Let m be a fixed positive integer in Z. Define the relation $\equiv_n$ on Z as follows for all $x, y \in Z$. $x \equiv_n y$ iff $\frac{n}{x-y}$ i.e $x - y = nk$. Show that $\equiv_n$ is an equivalence relation in Z. **(6 Marks)**

b) Let $f : G \to G_1$ be a group homomorphism. Show that kernel of f is a normal subgroup of G.
(8 Marks)

c) Show that every subgroup of an abelian group is normal **(6 marks)**

QUESTION FOUR: (20 MARKS)

a) Define normal subgroup and prove that every subgroup of index 2 is normal. **(6 Marks)**

b) Let H be a normal subgroup of G. Denote the set of all left cosets $\{aH \mid a \in G\}$ by $\frac{G}{H}$ and define * in $\frac{G}{H}$ for all $aH, bH \in \frac{G}{H}$ by $(aH)*(bH) = abH.$ Show $\left(\frac{G}{H}, *\right)$ is a group **(8 Marks)**

c) Let R_1 and R_2 be subrings of R .Show that $R_1 \cap R_2$ is a subring of $R.$ **(6 Marks)**

QUESTION FIVE: (20 MARKS)

a) State the Lagrange's Theorem **(3 Marks)**

b) List all the elements of a symmetric group of order 3 and construct a multiplication table for S_3 **(12 Marks)**

c) Prove that any two, right and left cosets of H in G are disjoint. **(5 Marks)**