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KIRIRI WOMEN'S UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
SECOND YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN
MATHEMATICS
KMA 303- INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS

Date: 19TH APRIL 2023

Time: 2:30 PM-4:30PM

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

- a) Solve the differential equation;

$$\frac{dy}{dx} = \frac{2x-2y+3}{x-y+1} \quad (3 \text{ Marks})$$

- b) The acceleration of a particle is given by $a = 6t + 10 \text{ cm/sec}^2$. If the initial velocity is 3 cm/sec what is the velocity of the particle at $t = 2 \text{ sec}$. (4 Marks)

- c) Show that $3x(xy - 2)dx + (x^3 + 2y)dy = 0$ is exact. (4 Marks)

- d) Using separation of variable technique to show $(y - 2)^2 dx - x^2 dy = 0$ (4 Marks)

- e) Prove that $y = e^{2x}$, is a solution to the differential equation $y'' + y' - 6y = 0$ (4 Marks)

- f) Eliminate the constants from the equation $(x - a)^2 + y^2 = a^2$ (3 Marks)

- g) Solve the DE $\frac{\partial y}{\partial x} = \frac{2y}{x}$ for $y > 0$ and $x > 0$ (4 Marks)

- h) Given the $y = ce^{-x} + de^{-2x}$ and $y(0) = 1$ and $y'(0) = 2$, obtain the particular solution. (4 Marks)

QUESTION TWO (20 MARKS)

- a) Determine if the following differential equations are homogeneous;

i) $y' = \frac{y+x}{x}$ (3 Marks)

ii) $y' = \frac{y+x^3}{x^2}$ (3 Marks)

- b) Bone remains in an ancient excavation site contain only 14% of the Carbon-14 found in living animals today. Estimate how old the bone remains are. Use the fact that the half-life of the Carbon-14 is $\tau = 5730$ years. (8 Marks)

- c) Solve the differential equation $y'' - y' - 2y = 0$ (6 Marks)

QUESTION THREE (20 MARKS)

- a) Find the orthogonal trajectory of the family of curves $x^2 - y^2 = Cx$ (4 Marks)
- b) Find the differential equation associated to the primitive $y = [A \cos mx + B \sin mx]e^{kx}$. Determine the order, degree and linearity of the differential equation. (5 Marks)
- c) Detectives discover a murder victim at 5:40 a.m. and the body temperature of the victim is 27°C . Thirty minutes later, the police surgeon arrives and the body temperature is 23°C . If the air temperature is 16°C and the normal body temperature is 37°C , at what time did the victim die? (6 Marks)
- d) Use the method of undetermined coefficient to solve;
 $y'' - 2y' = e^x \sin x$ (5 Marks)

QUESTION FOUR (20 MARKS)

- a) Using the substitution $Z = \frac{dy}{dx}$, solve the initial value problem;
 $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 2xe^{2x}$, given $y(0) = -\frac{1}{2}$, and $y'(0) = -5$ (5 Marks)
- b) Find the general solution of the differential equation $2y'' - 3y' + y = 0$ (5 Marks)
- c) Solve the Bernoulli's differential equation $\frac{dy}{dx} + xy = xe^{-x^2}y^{-3}$ (5 Marks)
- d) Solve the differential equation;
 $(D^4 + 8D^2 + 16)y = 0$ (5 Marks)

QUESTION FIVE (20 MARKS)

- a) A person places Ksh. 20000 in a savings account which pays 5% interest per annum, compounded continuously. Find;
- i) The amount in the account after 3 years. (7 Marks)
- ii) The time required for the amount to double in value, presuming no withdrawals or deposits are made. (4 Marks)
- b) Prove that $(3x^4y + x)\partial x + x^5\partial y = 0$ is not exact and hence use integrating factor to make it exact. (9 Marks)