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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
FIRST YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN COMPUTER SCIENCE

KMA 2105 DISCRETE MATHEMATICS

Date: 7TH AUGUST, 2024

Time: 11:30 AM – 1:30 PM

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

- a) State the truth values of the following:
- i. $\forall x \in \mathbb{R}, x + x \geq x$ (1 Mark)
 - ii. $\forall x \in \mathbb{N}, x + x > x$ (1 Mark)
 - iii. $\exists x \in \mathbb{N}, 2x + 3 = 6x + 7$ (1 Mark)
- b) Prove using contradiction that if n is an integer and that $3n + 2$ is odd, then n is odd. (3 Marks)
- c) Let $A = \{1, 3, 7, 5\}$, $B = \{a, b, 1, 7, 4\}$. Evaluate $B - A$ (3 Marks)
- d) Let A and B be sets, by use of Venn diagrams show that $\sim(A \cap B) = \sim A \cup \sim B$ (4 Marks)
- e) Let $f, g: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 + 1$ and $g(x) = 2x - 5$. Find
- i. $(g \circ f)(x)$ (2 Marks)
 - ii. $(f \circ g)(2)$ (3 Marks)
- f) Given $A = \{2, 3\}$ and $B = \{a, \{b, c\}, d\}$ find $A \times B$ and $B \times A$ (3 Marks)
- g) Write the inverse, converse and contrapositive of "If cows can dance, then $6+3 = 3$ " (3 Marks)
- h) Find the sets A and B given that $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$ and $A \cap B = \{3, 6, 9\}$ (3 Marks)
- i) Show that the proposition $p \rightarrow q$ and $\sim p \vee q$ are logically equivalent using truth tables (3 Marks)

QUESTION TWO: (20 MARKS)

- a) Prove using mathematical induction that the sum of the first n odd numbers is n^2 (5 Marks)
- b) Test the validity of the given argument (5 Marks)

$$\begin{array}{l} p \vee q \\ p \rightarrow r \\ \underline{q \rightarrow r} \\ \therefore r \end{array}$$

- c) Construct a truth table for the compound proposition $(p \rightarrow q) \wedge [(q \wedge \sim r) \rightarrow (p \vee r)]$ (5 Marks)
- d) Out of 30 KWUST football fans, 11 support Manchester City, 9 support Arsenal, 17 support Chelsea, 8 support Manchester United. 4 support Manchester City and Arsenal, 5 support Manchester City and Chelsea, 5 support Arsenal and Chelsea, 6 support Chelsea and Manchester United, the number of those who support Chelsea and Manchester United is twice those who support Manchester City, Arsenal and Chelsea, 2 support none of these teams.
- Express this information on a Venn diagram. (3 Marks)
 - Find how many supports at most two teams (1 Mark)
 - Find how many supports at least two teams (1 Mark)

QUESTION THREE: (20 MARKS)

- a) In a survey of 500 people, the following information was gathered regarding their interest in sports games: 285 are interested in football, 195 are interested in hockey, and 115 are interested in basketball. Furthermore, 45 people are interested in both football and basketball, 70 people are interested in both football and hockey, and 50 people are interested in both hockey and basketball. However, 50 people are not interested in any of the three games. Determine the number of people who are interested in:
- all three games. (3 Marks)
 - exactly one of the games. (3 Marks)
 - exactly two of the games. (3 Marks)
 - at least one of the games. (3 Marks)
- b) Using Mathematical induction prove that
- $$1^2 + 2^2 + 3^2 + \dots + (2n - 1)^2 = \frac{1}{3}n(4n^2 - 1) \quad (6 \text{ Marks})$$
- c) Find the domain and range of the real valued function $g(x) = \sqrt{16 - x^2}$ (2 Marks)

QUESTION FOUR: (20 MARKS)

- a) Prove that $\sqrt{2}$ is irrational using contradiction (5 Marks)
- b) Use truth tables to determine whether the following compound proposition is a tautology, a contradiction or a contingency
- $$\sim(p \vee (q \rightarrow r)) \leftrightarrow ((p \leftrightarrow q) \wedge r) \quad (8 \text{ Marks})$$
- c) Determine the power set given the set $A = \{a, b, c, d\}$ (4 Marks)
- d) Let $g: \mathbb{Z} \rightarrow \mathbb{Z}$ be defined by $g(x) = 3x + 2$, determine if g is invertible (3 Marks)

QUESTION FIVE: (20 MARKS)

- a) Show that the product of any two rational numbers is rational (4 Marks)
- b) Use mathematical induction to prove that $8^n - 1$ is divisible by 7 for all positive integers n (6 Marks)
- c) Construct a truth table for the following compound propositions
- $(p \vee q) \wedge \sim r$ (4 Marks)
 - $p \rightarrow q \wedge [(p \vee \sim r) \rightarrow (q \wedge r)]$ (6 Marks)