

# **KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR** FIRST YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (COMPUTER SCIENCE)

## **KMA 2106: PROBABILITY AND STATISTICS I** DATE: 6<sup>TH</sup> DECEMBER 2024 TIME: 8:30AM - 10:30AM

### **INSTRUCTIONS TO CANDIDATES** ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

## **QUESTION ONE: COMPULSORY (30 MARKS)**

a) A random variable X has a probability distribution given by

$$f(x) = \begin{cases} k (1-x), & 0 \le x \le 1 \\ 0, & Otherwise \end{cases}$$

i) Find the value of the constant k.

ii) Evaluate the 
$$P\left(X \le \frac{1}{3}\right)$$
.

b) For a random variable X with p.m.f given by

x	0	1	2	3	4	5
P(X=x)	0.1681	0.3601	0.3087	0.1323	0.0284	0.0024

determine the;

i)	mode.	(2 Marks)
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- ii) Upper and Lower quartiles.
- c) A random variable X has a pdf given by

$$f(x) = \begin{cases} 3 e^{-3x}, & x > 0\\ 0, & Otherwise \end{cases}$$

I) Find the moment generating function of X.

II) Use the moment generating function obtained in i) to find

- i) Mean of X. (2 Marks)
- ii) Variance of X. (3 Marks)
- d) On average, one out of 10 telephones numbers are found to be busy. Six telephone numbers are selected at random. Let X be the number of busy telephone numbers out of the six selected ones.
  - i) Write down the probability distribution of X. (1 Marks)
  - ii) Find the probability that at most two of them will be busy. (3 Marks)



(3 Marks)

(2 Marks)

(3 Marks)

(3 Marks)

- e) The final exam scores in a statistics class were normally distributed with a mean of 63 and a standard deviation of 5. Find the probability that a randomly selected student score between 60 and 70.
  (3 Marks)
- f) A construction firm has placed an order that they require a consignment of wires which have a mean length of 10.5 meters with a standard deviation of 1.7 m. The company which produces the wires delivered 90 wires, which had a mean length of 9.2 m. The construction company rejected the consignment on the grounds that they were different from the order placed. Conduct a statistical test to indicate whether you support or not support the action taken by the construction company at 5% level of significance. (5 Marks)

## **QUESTION TWO: (20 MARKS)**

a) Let X be a continuous random variable with a geometric distribution given by

$$f(x) = \begin{cases} \theta \ e^{-\theta x}, & x > 0 \\ 0, & Otherwise \end{cases}.$$

Without using moment generating function, show that

i) 
$$E(X) = \frac{1}{\theta}$$
. (4 Marks)

ii) 
$$Var(X) = \frac{1}{\theta^2}$$
. (6 Marks)

b) The length of time a computer battery lasts from the time it is manufactured is known to be exponentially distributed with a mean life of 10 years. What is the probability that the battery last for;

i) Less than 5 years.	(3 Marks)
ii) Between 7 and 12 years.	(3 Marks)
iii) 3 more years given that it has lasted for 4 years.	(4 Marks)

## **QUESTION THREE: (20 MARKS)**

a) Consider a binomial random variable X with probability distribution function given by

$$f(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, x = 0, 1, \dots, n\\ 0, \text{ elsewhere} \end{cases}$$

- I) Determine the moment generating function of X. (4 Marks)
- II) Use the mgf above to obtain;

- ii) the variance. (3 Marks)
- b) Suppose 40% of the population approves of the job the governor is doing, and that 20 individuals are drawn at random from the population. What is the probability that;
  - i) exactly 7 people will support the governor? (2 Marks)
  - ii) fewer than 5 people will support the governor? (3 Marks)
- c) The number of tasks send to a company printer in busy time is known to be Poisson distributed with parameter  $\lambda = 2$  per minute. Compute the probability that;
  - i) At least 2 tasks are sent in a minute. **3 Marks**)
  - ii) Not more than three tasks are sent in three minutes. (3 Marks)

## **QUESTION FOUR: (20 MARKS)**

a)	Medical researchers have found out that the length of human pregnancy is	s normally				
	distributed with mean of 40 weeks and a standard deviation of 2 weeks.					
I) Of 500 pregnant women, how many would you expect their pregnancy to la						
	i) Less than 37 weeks?	(3 Marks)				
	ii) Between 38 and 41 weeks?	(3 Marks)				
	II) If 90% of women have pregnancies not lasting for more than K weeks, find the	ne value of				
	К.	(3 Marks)				
b)	The amount of time, in minutes, that a person must wait for a bus is uniformly	distributed				
	between 0 and 15 minutes, inclusive. What is the probability that;					

- i) a person waits fewer than 12.5 minutes? (2 Marks)
- ii) the waiting time will be between 0.5 standard deviations from the mean. (4 Marks) c) A school has 20 students in a class, 8 of whom are girls and 12 are boys. The teacher wants
  - to select a group of 5 students to participate in a special project. What is the probability that;
    - i) exactly 3 girls will be chosen in this group of 5? (2 Marks)
    - ii) at least 2 girls will be chosen in this group of 5? (3 Marks)

# **QUESTION FIVE: (20 MARKS)**

- a) Highlight the steps in hypothesis testing.
- b) A simple random sample of 10 people from a certain population has a mean age of 27. Can we conclude that the mean age of the population is less than 30? The variance is known to be 20. Let  $\alpha = .05$ . (5 Marks)
- c) The average cost (in hundreds of shillings) of a hotel room in Nairobi is said to be 168 per night. To determine if this is true, a random sample of 25 hotels is taken and resulted in  $\bar{x} = 172.5$  and s = 15.40.
  - i) Test the appropriate hypotheses at  $\alpha = 0.05$ .
  - ii) Compute 95% confidence intervals of the average cost of hotel rooms in Nairobi.

## (4 Marks)

#### (6 Marks)

#### (5 Marks)