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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
FIRST YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF EDUCATION(ARTS)

KMA 2102 DIFFERENTIAL CALCULUS

Date: 12th AUGUST 2024
Time: 8.30am-10.30am

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS
QUESTION ONE: COMPULSORY (30 MARKS)

a) Evaluate the limits of the following functions

i. $\lim_{x\to\infty} \sqrt{x^2-4x}-x$ **(5 Marks)**

ii. $\lim_{x\to 1}\left(\frac{x^3-1}{x^2-1}\right)$ **(3 Marks)**

b) Find the value of A and B so that the following function is continuous for all x

$$f(x)=\begin{cases} A\left(\frac{1-\cos x}{sin^2 x}\right) & if\ x<0 \\ 2x^2-x+B\ if\ 0\le x\le 1 \\ \frac{x^2+2x-3}{x^2-1} & if\ x>1 \end{cases}$$ **(3 Marks)**

c) Use first principle of differentiation to find $\frac{dy}{dx}$ given that $y(x)=\frac{1}{x}$ **(4 Marks)**

d) A car travels along a straight road at a constant velocity of 60 km/h. How far does the car travel in 2.5 hours? **(3 Marks)**

e) If $y=\sin^{-1}(\cos x)$, show that $\frac{dy}{dx}=-1$ **(4 Marks)**

f) Find $\frac{dy}{dx}$ given that

i. $y=x^{tan(x^2)}$ **(4 Marks)**

ii. $x=3t^2-4t+6$ and $y=16-t^4$ **(4 Marks)**

QUESTION TWO: (20 MARKS)

a) If $y=(\tan x+\sec x)^m$, where m is a positive integer.
Show that $\frac{dy}{dx}=mysec\ x$ **(6 Marks)**

b) Explain what is meant by "a function $f(x)$ is continuous at an interior point $x=a$". Hence, determine the values of constants A and B so that the following function is continuous everywhere on the real number line.

$$f(x)=\begin{cases} 2, & if\ x<1 \\ Ax+B, & if\ 1\le x\le 2 \\ 6, & if\ x>2 \end{cases}$$ **(8 Marks)**

c) Find the derivative of the function represented by $x = \frac{2t}{1+3t}$ and $y = t^3 - 4t + 8$ **(6 Marks)**

QUESTION THREE (20 MARKS)

a) If $y = (tan^{-1}x)^2$, prove that $\frac{d}{dx}\left\{(1+x^2)\frac{dy}{dx}\right\} = \frac{2}{1+x^2}$ **(4 Marks)**

b) Determine the co-ordinates and the nature of the turning points on the curve defined by $y = 2x^3 + 9x^2 - 60x$ **(6 Marks)**

c) If y is a differentiable function of x, find the derivative of y with respect to x given that $y = x^3 e^{x^3+4\sqrt{x}} - cos(ln2x)$ **(5 Marks)**

d) In marketing a certain item, a business has discovered that the demand for the item is represented by $p(x) = \frac{50}{\sqrt{x}}$. The cost of producing x items is given by $c(x) = \frac{1}{2}x + 500$. Find the price per unit that will yield maximum profit. **(5 Marks)**

QUESTION FOUR: (20 MARKS)

a) By the use of power rule, find the third derivative of the following function $y = 3x^4 + 2x^3 + x + 7$ **(3 Marks)**

b) Find the equation of the normal to the curve $x^2 + 2xy + 3y^2 = 1$ at point $(2,1)$ **(5 Marks)**

c) A rectangular box is to be made from a piece of cardboard measuring 24 inches long by 9 inches wide by cutting out identical squares from the four corners and turning up the sides. Find the dimensions of the box that will maximize the volume. What is the maximum volume? **(7 Marks)**

d) Evaluate $\lim_{x\to 5}\left(\frac{|x-5|}{(x-5)}\right)$ **(5 Marks)**

QUESTION FIVE: (20 MARKS)

a) Use first principle to find $\frac{dy}{dx}$ given $y(x) = \cos x$ **(5 Marks)**

b) Classify the extreme points of $y(x) = 2x^3 + 3x^2 - 12x$ **(4 Marks)**

c) A manufacturer wants to design an open box having a square base and surface area of $108m^2$. Find the dimensions of the box that will give maximum volume. **(7 Marks)**

d) Find $\frac{dy}{dx}$ given the following function $y = \frac{2-x^2}{1+3x^2}$ **(4 Marks)**