Kasarani Campus
Off Thika Road
P. O. Box 49274, 00101
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMEN'S UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2022/2023 ACADEMIC YEAR
FOURTH YEAR, SECOND SEMESTER, EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

KMA 413 - INTRODUCTION TO STOCHASTIC PROCESSES

Date: 20th April 2022
Time:8.30am-10.30am

INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Show that $\frac{1}{1-s}$ is the generating function of {1, 1, 1, ……} hence find the generating function of the sequence $\{1^2, 2^2, 3^2, 4^2, ...\}$ [6 marks]

b) Let X and Y be independent Poisson distribution variable with parameters λ_1 and λ_2 respectively. Show that the sum of the two Poisson random variables with parameters λ_1 and λ_2 is a Poisson variable with parameters $\lambda_1 + \lambda_2$ [6 marks]

c) Distinguish between a stationary and evolutionary stochastic process [2 marks]

d) Using the probability generating function technique, show that

i) E(X)=$G'(1)$ [2 marks]

ii) $V(X) = G''(1) + G'(1) - [G'(1)]^2$ [4 marks]

e) Consider a Markov chain with two states and transitional probability matrix given by

$$P=\begin{pmatrix} 0.5 & 0.5 \\ 0.3 & 0.7 \end{pmatrix}$$

Determine the stationary distribution of the chain [6 marks]

f) Suppose X has a geometric distribution with parameter P. Determine the probability generating function of X and hence find the mean and variance of X.

[6 marks]

QUESTION TWO (20 MARKS)

a) Given that $A(s) = \frac{1}{(s-2)(s-3)^2}$. Determine a_8 [8 marks]

b) Given that $S_N = X_1 + X_2 + \cdots + X_N$ where X_i and N are random variables:

i) Find the PGF of S_N [4 marks]

ii) Using (i) determine the $E(S_N)$ and $\text{Var}(S_N)$ [8 marks

QUESTION THREE (20 MARKS)

a) Given that P is given by;

$$P=\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

i) Show that the state E_1 is a periodic. [4 marks]

ii) Given that the Markov chain is irreducible, show that exists

$$\lim_{n\to\infty} P_{ij}^{(n)} = \pi_j \text{ exists}$$

[7 marks]

b) In a certain process the probability of *n* off springs from one ancestor is geometric with parameter *p*

i) Determine the range of values for which the process will die out with probability I [3 marks]

ii) For P outside this range, find the probability of extinction [3 marks]

iii) If P is chosen so that the probability or process never dies out is 0.999, what is the probability that an individual will have an offspring. [3 marks]

QUESTION FOUR (20 MARKS)

A continuous-time Markov sickness and death model has four states: *H* (healthy), *S* (sick), *T* (terminally ill) and *D* (dead). From a healthy state transitions are possible to states namely *S* and *D*, each at rate 0.05 per year. A sick person recovers his health at rate 1.0 per year; other possible transitions are to *D* and *T*, each with rate 0.1 per year. Only one transition is possible from the terminally ill state, and that is to state *D* with transition rate 0.4 per year.

i) Draw the transition graph for this process. [3 marks]

ii) Define $P(t) = \{p_{ij}(t): i, j \in H, S, T, D\}$ where $p_{ij}(t)$ denotes the probability of being in state *j* at time *t* given that the individual was in state *i* at time 0. State the Kolmogorov forward equation satisfied by the matrix *P*(*t*), making sure that you specify the entries of the matrix *A* which appears. [4 marks]

iii) Calculate the probability of being healthy for at least 10 uninterrupted years given that you are healthy now. [2 marks]

iv) Let d_j denote the probability that a life which is currently in state *j* will never suffer a terminal illness. By considering the first transition from state *H*, show that $d_H = \frac{1}{2} + \frac{1}{2} d_s$ and deduce similarly that $d_s = \frac{1}{12} + \frac{5}{6} d_H$, Hence evaluate d_H and d_s . [6 marks]

v) Write down the expected duration of a terminal illness, starting from the moment of the first transition into state *T*. Use the result of (iv) to deduce the expectation of the future time spent terminally ill by an individual who is currently healthy. [5 marks]

QUESTION FIVE (20 MARKS)

a) In Markov chain, when is a state said to be;

i) Recurrent [1 mark]

ii) Transient [1 mark]

iii) Ergodic [1 mark]

iv) A periodic [1 mark]

v) Convolution sequence. [2 marks]

b) State the assumption of birth and death processes. [6 marks]

c) Find the distribution of $S_N = X_1 + X_2 + \cdots + X_N$ where X is are independent random variables from a binomial distribution with parameters N_i and P for $i = 1, 2,3, \ldots, N$.

[12 marks]