

Kasarani Campus Off Thika Road P. O. Box 49274, 00101 NAIROBI Westlands Campus Pamstech House Woodvale Grove Tel. 4442212 Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR THIRD YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE(MATHEMATICS)

KMA 2301: NUMERICAL ANALYSIS I

DATE: 6TH DECEMBER 2024 TIME: 11:30AM-1:30PM

<u>INSTRUCTIONS TO CANDIDATES</u> ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

- a) Use Gauss Chebyshev 3-point formula to evaluate $\int_{-1}^{1} (1-x^2)^{\frac{3}{2}} \cos x \, dx$ correct to 5 decimal places. (4 Marks)
- b) Find the cubic polynomial which takes the following values using Newton's forward interpolation formula (5 Marks)

Х	0	1	2	3
f(x)	1	2	1	10

- c) Use method of synthetic division to obtain the quotient and the remainder when the polynomial $3x^5 + 5x^4 + 8x^2 + 7x + 4$ is divided by (x+2) (2 Marks)
- d) Find the appropriate value of $I = \int_0^1 \frac{1}{1+x} dx$ using trapezoidal rule by taking n=5. Obtain the bound for truncation error. (5 Marks)

e) Find an efficient method for computing square roots based on the Newton's method. Hence compute $\sqrt{17}$ correct to six decimal places. Perform only 3 iterations. (5 Marks)

f) Determine the appropriate root of the equation $x - 2 \sin x = 0$ taking $x_0 = 2$ and using Aitken's δ^2 -process to 2 d.p. (5 Marks)

g) Evaluate
$$x^4 - 8x^3 + 5x^2 + 7x - 7$$
 at $x = 2 + i3$ (4 Marks)

QUESTION TWO: (20 MARKS)

a) Using quadratic Lagrange interpolation and given that y(0) = 15, y(1) = 48, y(5) = 85. Find

- i. The Lagrange interpolating polynomial $P_2(x)$ (5 Marks)
- ii. Value of y at x = 2 (2 Marks)
- b) Evaluate $\int_{4}^{5.2} \log e^x dx$ given that h = 0.2 using
 - i. Trapezoidal rule

(3 Marks)

	ii.	Simpson rule	(3 Marks)
	iii.	Simpson $\frac{3}{8}$ rule	(3 Marks)
c)	Us	e synthetic division to obtain quotient and remainder if $(4x^4 + 5x^3 + 3x^2 + 3x^2)$	+ 6 <i>x</i> – 9) ÷
	(2:	$x^2 + 4x - 6)$	(4 Marks)
Q	UE	STION THREE: (20 MARKS)	
a)	Est	timate $\int_0^1 \frac{1}{1+x} dx$ correct to 6 decimal places using Gauss Legendre	
	i.	2-point formula	(4 Marks)
	ii.	3-point formula	(4 Marks)
b)	Us	ing Bisection method to approximate the root of $f(x) = x^3 + x^2 - 3x - 3$	
	i.	Show that $f(x)$ has a root between 1 and 2	(2 Marks)
	ii.	Determine the minimum number of iterations which can be performed for	the root of the
		equation $f(x) = 0$ accurate to 2 decimal places.	(4 Marks)
	iii.	Find the approximate root correct to 2 decimal places.	(6 Marks)

QUESTION FOUR: (20 MARKS)

a) From the table below, estimate the number of students who obtained marks between 40 and 45 using Newton's forward interpolation formula (6 Marks)

1						
Marks	30-40	40-50	50-60	60-70	70-80	
No. of students	31	42	51	35	31	
				n		

b) Given that $\overline{f(x)} = 2x^3 - 7x + 2$ and using the iterative formula $x_{n+1} = \frac{2}{7}(x_n^3 + 1)$

i.	Show that $f(x)$ has a root between 0 and 1	(2 Marks)
ii.	Determine if the iterative x_{n+1} converges.	(3 Marks)

iii. Hence find the root of f(x) correct to 3 decimal places. (5 Marks)

c) Estimate $\int_{1}^{1.3} \sqrt{x} \, dx$ to 4 decimal places using Simpson's $\frac{1}{3}$ rule and when h = 0.15

(4 Marks)

QUESTION FIVE: (20 MARKS)

a) Use Newton's iterative with synthetic division to find a zero of the polynomial f(x) = x⁴ - 4x³ + 7x² - 5x - 2 starting with x₀ = 0 correct to 4 decimal places. Perform 5 iterations. (6 Marks)
b) In determining the approximations to the positive roots of the equation 2x³ - 7x + 2 = 0 using the rule of false position in 3 decimal places

Investigate the convergence of the iterative.

(4 Marks)

- ii. Hence perform 4 iterates to approximate the root (5 Marks)
- c) Using the data below, find Newton's interpolating polynomial (5 Marks)

X	0	10	20	30	40
у	7	18	32	51	87