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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2022/2023 ACADEMIC YEAR
FIRST YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS AND COMPUTER SCIENCE)

Date: 13th April, 2022
Time: 8.30am – 10.30am

KMA 105 - DISCRETE MATHEMATICS

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

- a) List the members of these sets
- i) $\{x | 5 \leq x \leq 30 \text{ and } x \text{ is a sexy prime}\}$
 - ii) $\{x | x \text{ is a real number such that } x^2 = 1\}$ (4 marks)
- b) Write the inverse, converse and contrapositive of the given statement “If Kenya can qualify for AFCON, then Kenya can finish third in the competition” (3 marks)
- c) Suppose that A is the set of sophomores at your school and B is the set of students in discrete mathematics at your school. Express each of these sets in terms of A and B .
- i) the set of sophomores taking discrete mathematics in your school (1 mark)
 - ii) the set of sophomores at your school who are not taking discrete mathematics (1 mark)
 - iii) the set of students at your school who either are sophomores or are taking discrete mathematics (1 mark)
 - iv) the set of students at your school who either are not sophomores or are not taking discrete mathematics (1 marks)
- d) If $A = \{a, b, c, d\}$ and $B = \{1, 2, 3, 8, 5\}$ find $A \times B$ and $|A \times B|$ (4 marks)
- e) Let $f, g: \mathbb{R} \rightarrow \mathbb{R}$ be the function with the rule $f(x) = \frac{2}{3}x - 2$ and $g(x) = \frac{5}{2}x + 5$. Find $f \circ g$ and $g \circ f$ (4 marks)
- f) Prove that if n is an integer and $3n + 2$ is odd, then n is odd using a proof by contraposition (3 marks)
- g) Using set identities show that for any two sets $A - B = A \cap B^c$ (4 marks)
- h) Let p and q be the propositions p : I played in AFCON for the first time.
 q : I won the AFCON.
Express proposition $\neg p \vee (p \wedge q)$ as an English sentence. (4 marks)

QUESTION TWO (20 MARKS)

- a) Express the negations of the following propositions using quantifiers and in English
- i) There is a student in this class who has never seen a computer
 - ii) Every student in this class likes mathematics
 - iii) There is a student in this class who has been in at least one room of every building on campus (3 marks)
- b) Given that $f(x) = 2x$, $g(x) = x^2$ and $h(x) = x + 1$, find:
- i) $f \circ (g \circ h)$
 - ii) $g \circ (f \circ h)$ (4 marks)
- c) A survey on a sample of 25 new cars being sold at a local auto dealer was conducted to see which of three popular options, air-conditioning (A), radio (R), and power windows (W), were already installed. The survey found: 15 had air-conditioning (A), 5 had A and W , 12 had radio (R), 9 had A and R , 3 had all three options. 11 had power windows (W), 4 had R and W . Represent this information in a well labelled Venn diagram and hence find the number of cars that had:
- i) only W (2 marks)
 - ii) R and W but not A (2 marks)
 - iii) only one of the options (2 marks)
 - iv) none of the options (2 marks)
- d) Construct a truth table to show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent. (5 marks)

QUESTION THREE (20MARKS)

- a) Use a direct proof to show that if n is an even integer, then 4 divides n^2 (4 marks)
- b) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 2x - 3$. Find f^{-1} (4 marks)
- c) Determine the power set $P(A)$ of $A = \{a, b, c, d\}$. (4 marks)
- d) Using a Venn diagram to show that $\overline{A \cup B} = \bar{A} \cap \bar{B}$, if A and B are sets (4 marks)
- e) Use mathematical induction to prove that $1 + 2 + 2^2 + 2^3 + \dots + 2^n = 2^{n+1} - 1$ (4 marks)

QUESTION FOUR (20 MARKS)

- a) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 7x^2 + 1$ and $g(x) = x^3 - 2$. Find the formula for the composition functions $g \circ f$, $f \circ g$ and $f \circ f$ (6 marks)
- b) Show that for any two sets $A - B = A \cap B^c$ using a Venn diagram (3 marks)
- c) Prove that \sqrt{p} is irrational by contradiction (7 marks)
- d) Show that the propositions $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent. (4 marks)

QUESTION FIVE (20 MARKS)

- a) Distinguish between a tautology and a contradiction (3 marks)
- b) Use mathematical induction to prove that $12^n - 1$ is divisible by 11, $\forall n \in \mathbb{N}$. (7 marks)
- c) Find the number of integers between 1 and 100 inclusively that are divisible by either 3, 5 or 7 (5 marks)
- d) Let p and q denote: “I do not study”, and “I fail” respectively. State the verbal translation of each of the following
- i) $p \wedge q$
 - ii) $\neg p \vee q$
 - iii) $\neg p \wedge \neg q$
 - iv) $\neg(p \vee \neg q)$
 - v) $\neg(\neg p \vee \neg q)$ (5 marks)