# KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
## UNIVERSITY EXAMINATION, 2022/2023 ACADEMIC YEAR
## SECOND YEAR, FIRST SEMESTER EXAMINATION
## FOR THE DEGREE OF BACHELOR OF BUSINESS INFORMATION TECHNOLOGY

Date:  29th July, 2022
Time:  8.30am –10.30am

## KMA 2208  -  PROBABILITY AND STATISTICS 1

**INSTRUCTIONS TO CANDIDATES**

ANSWER **QUESTION ONE  (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

**QUESTION ONE     (30 MARKS)**

a)      Differentiate between discrete and continuous random variable. Give appropriate examples
(4 Marks)
b)      State two properties of a sampling distribution.                                                                (2 Marks)
c)      Let $Z$ be a standard normal random variable. Calculate  $P(|Z| \le 2.50)$                        (3 Marks)
d)      Let $X$ be a random variable defined the function

$$f(x) = \begin{cases} 3x^2, & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$$

Find the mean of $X$                                                                                                        (4 Marks)
e)      The length in time for one individual to be served at a cafeteria is a random variable having the
exponential distribution with mean of 4 minutes. What is the probability that a person is served in
less than three minutes?                                                                                                    (6 Marks)
f)      Outline the procedure for testing hypothesis                                                                (5 Marks)
g)      A population distribution is known to have standard deviation 20. Test the claim that the population
mean is equal to 50, if the average of a sample of 64 observations is 55. Test at $\alpha = 0.05$
(6 Marks)

**QUESTION TWO    (20 MARKS)**
a)      State two condition that a function of a continuous random variable should satisfy to be probability
density function                                                                                                                (2 Marks)
b)      The grade average points (GPA's) for grading students at KWUST are distributed as a continuous
random variable $X$ with probability density function

$$f(x) = \begin{cases} k(1 - (x-3)^2), & 2 \le X \le 4 \\ 0, & Otherwise \end{cases}$$

 Find the
i)       value of k                                                                                                                (3 Marks)
ii)      probability that a GPA exceeds 3                                                                            (3 Marks)
iii)     probability that a GPA is between 1.8 and 3.9                                                        (4 Marks)
iv)      mean of $X$                                                                                                            (3 Marks)
v)       variance of $X$                                                                                                        (5 Marks)

## QUESTION THREE(20 MARKS)

a) It is expected that 10% of production from a continuous process would be defective. Find the probability that in a sample of 10 units chosen at random

      i) Write down the probability of X, given that X denotes the number defective    (1 Mark)

      ii) Exactly 2 will be defective

                                                (3 Marks)

      iii) At least 2 will be defective                                  (3 Marks)

b) The number of industrial injuries per working week in a particular factory is known to follow a Poisson distribution with mean 0.5. Find the probability that:

      i) In a particular week there will be

            a) Less than two accidents                                (4 Marks)

            b) At least two accidents                                (3 Marks)

      ii) In a three-week period, there will be no accident            (2 Marks)

c) Bolts are manufactured by a machine and it is known that approximately 20% are outside certain tolerance limits. If a random sample of 200 is taken, find the probability that 50 bolts will be outside the limits.

                                                             (4 Marks)

## QUESTION FOUR   (20 MARKS)

a) State the Central Limit Theorem                                    (2 Marks)

b) KPMG Corporation gives each of its employees an aptitude test. The scores on the test are normally distributed with a mean of 75 and a standard deviation of 15. A simple random sample of 25 is taken from a population of 500.

      i) What is the probability that the average aptitude test score in the sample will be between 70.14 and 82.14?                                          (4 Marks)

      ii) What is the probability    v

      iii)

      iv)

        that the average aptitude test score in the sample will be equal to or greater than 82.68?

                                               (4 Marks)

      v) Find a value, C, such that $P\left( \bar{X} \geq C \right) = 0.015$                    (4 Marks)

c) Let X be an exponential random variable given by probability density function

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & 0 < x < \infty \\ 0, & elsewhere \end{cases}$$

  If $\Pr(X \leq 2) = \Pr(X > 2)$, find $E[X]$ and $Var(X)$                    (10 Marks)

## QUESTION FIVE    (20 MARKS)

a) Differentiate between the following terms:

      i) Null hypothesis and alternative hypothesis                              (2 Marks)

      ii) Type I error and Type II error                                    (2 Marks)

      iii) One-sided tests and Two-sided tests                              (2 Marks)

b) The claim is made that 40% of tax filers use computers software to file their taxes. In a sample of 50, 14 used computers to file their taxes. Test $H_0 : p = 0.4$ versus $H_1 : p < 0.4$ at $\alpha = 0.05$ where p is the population proportion who use computer software to file their taxes.      (6 Marks)

c) Suppose we would like to determine if the typical amount spent per customer for dinner at a new restaurant in town is more than Shs. 2000. A sample of 49 customers over a three-week period was randomly selected and the average amount spent was Shs. 2260. Assume that the standard deviation is known to be Shs. 250. Using a 0.05 level of significance, would we conclude the

typical amount spent per customer is more than Shs. 2000?