



Kasarani Campus
Off Thika Road
P. O. Box 49274, 00101
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2023/2024 ACADEMIC YEAR
END OF SEMESTER EXAMINATIONS
FOR THE DEGREE OF BACHELOR OF EDUCATION (ARTS)
KMA 2413 STOCHASTIC PROCESS

Date: APRIL 2024

Time: 2 Hrs.

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE COMPULSORY (30 MARKS)

- a) Consider a Fibonacci sequence given by $a_n = a_{n-1} + a_{n-2}$, $n \geq 2$ with $a_0 = 0$, $a_1 = 1$.

Find the generating function of the sequence. (4 marks)

- b) Consider a sequence whose generating function is

$$A(s) = \frac{1}{(2-s)(1+s)}$$

Find the k^{th} term of the sequence. (5 marks)

- c) Consider a Poisson random variable X with probability distribution

$$P_k = P(X = k) = \begin{cases} \frac{\lambda^k e^{-\lambda}}{k!}, & k = 0, 1, 2, \dots \\ 0, & \text{Otherwise} \end{cases}$$

- i) Find the probability generating function. (3 marks)

- ii) Use the probability generating function to determine;

I) Mean of X . (2 marks)

II) Variance of X . (3 marks)

- d) Supplies Company distributes a product that has a constant annual demand rate of 20,000 units.

A unit of the product costs Ksh300. Ordering cost is Ksh 4,000 per order and holding cost is

Ksh. 45 per unit. The company has 300 working days per year and the lead time is 10 days. Determine Economic order quantity and reorder point. (5 marks)

- e) A tailor specializes in ladies' dresses. The number of customers approaching the tailor appear to be Poisson distributed with mean of 6 customers per hour. The tailors attend the customers on first-come-first-served basis and the customers wait if need be. The tailor attends to the customers at an average rate of 10 customers per hour with the service time exponentially distributed. Determine

- i) the probability that there are 5 customers waiting for service. (2 marks)
- ii) the length of time a customer is expected to wait in the queue. (2 marks)
- iii) the probability that a customer will wait for 15 minutes. (2 marks)

QUESTION TWO (20 MARKS)

- a) Given the transition probability

$$P = \begin{pmatrix} 1/4 & 0 & 3/4 \\ 3/4 & 1/4 & 0 \\ 0 & 3/4 & 1/4 \end{pmatrix}$$

Obtain its stationary distribution. (5 marks)

- b) Classify the state of the following transition probability matrix

$$\begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & \dots & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & \dots & 0 \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{3} & 0 & 0 & 0 & 0 & \dots & \frac{2}{3} \end{bmatrix}$$

(9 marks)

- c) Explain **ANY THREE** areas of application of Markov chain transition probabilities. (6 marks)

QUESTION THREE (20 MARKS)

- (a) Write down the difference differential equation of a Poisson process. (2 marks)
- (b) Show that the Lagrange's linear equation of the process above is

$$\frac{\partial G(s, t)}{\partial t} + 0 \frac{\partial G(s, t)}{\partial s} = -\lambda(1 - s)G(s, t)$$

(5 marks)

- (c) Given the initial conditions $P_n(0) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$, Determine the probability that the population is of size (n) at time (t), $P_n(t)$. (7 marks)
- (d) Obtain mean and variance of the process. (6 marks)

QUESTION FOUR (20 MARKS)

- a) The initial cost of a machine is Rs. 7100 and scrap value is Rs. 100. The maintenance costs found from experience are as follows:

Year	1	2	3	4	5	6	7	8
Maintenance	100	250	400	600	900	1200	1600	2000
Resale Value	800	700	600	500	400	300	200	100

When should the machine be replaced?

(9 marks)

- b) Suppose we have 4 webpages that contains links to each other. We call the pages A, B, C, D.
- From page A, 10% link to page A, 30% of people link to page B, 40% link to page C, and 20% link to page D
 - From page B, 50% of people link to page A and 50% link to page D
 - From page C, 10% of people link to page B, 70% link to page C, and 20% link to page D
 - From page D, 20% of people link to page A, 40% to page B, 10% to page C, and 30% link to page D
- i) Write the transition matrix. (3 marks)
- ii) Find the probability that a person viewing page C will link to page D, for the first time, after two links if the link can be repeated. (4 marks)
- iii) Find the probability that a person viewing page C will return to page back to page C for the first time in three steps if links cannot be repeated in between. (4 marks)

QUESTION FIVE (20 MARKS)

- (a) Briefly discuss FOUR queuing structures (4 marks)
- (b) The ABC Mechanics has six service men of whom can service a car in 30 minutes on an average. The company are receiving cars for repair at a single counter and then sent for service in a different mechanics. Cars arrive at a service station at an average rate of 4 per hour.

Assuming that cars arrivals are Poisson distribute and the service is exponentially distributed exponentially, determine

- (i) The probability that the system shall be idle. (4 marks)
- (ii) The probability that there shall be 8 cars in the service center. (3 marks)
- (iii) The expected number of cars waiting in the queue. (3 marks)
- (iv) The expected number of cars in the service. (2 marks)
- (v) The average waiting time in the queue. (2 marks)