Kasarani Campus
Off Thika Road
Tel. 2042692 / 3
P. O. Box 49274, 00100
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2022/2023 ACADEMIC YEAR
THIRD YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

Date: 12th April, 2022
Time: 11.30am – 1.30pm

KMA 300 - APPLICATIONS OF LINEAR ALGEBRA

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Find the equation of the plane in 3-space that passes through the points $(1,1,-3)$, $(1,-1,1)$ and $(0,-1,2)$ (4 marks)

b) Explain the meaning of saddle point as used in game theory and identify the saddle points in the following payoff matrices

(i) $\begin{pmatrix} 3 & 1 \\ -4 & 0 \end{pmatrix}$ (ii) $\begin{pmatrix} 30 & -50 & -5 \\ 60 & 90 & 75 \\ -10 & 60 & -30 \end{pmatrix}$ (iii) $\begin{pmatrix} 0 & -3 & 5 & -9 \\ 15 & -8 & -2 & 10 \\ 7 & 10 & 6 & 9 \\ 6 & 11 & -3 & 2 \end{pmatrix}$ (6 marks)

c) An astronomer has discovered that an asteroid is moving in a circular path. If it passes through the three points (2, 6), (2, 0), (5, 3), obtain the equation of the circle and find center and radius of the circle. (7 marks)

d) Suppose that the oldest age attained by the females in a certain animal population is 15 years and that it is divided into three age classes and has a Leslie matrix $L = \begin{pmatrix} 0 & 4 & 3 \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \end{pmatrix}$. Beginning with the initial age distribution vector $x^{(0)} = \begin{pmatrix} 1000 \\ 1000 \\ 1000 \end{pmatrix}$. Find $x^{(1)}, x^{(2)}, x^{(3)}$.

(6 marks)

e) For the vertex matrix given, find all cliques in the corresponding directed graph

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

(6 marks)

QUESTION TWO (20 MARKS)

a) Find the equation of the plane passing through the three non-collinear points (1, 1, 0), (2, 0, −1), and (2, 9, 2) (4 marks)

b) A certain forest is divided into three height classes and has a growth matrix between harvests given by $G = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & \frac{2}{3} & 1 \end{pmatrix}$. If the price of trees in the second class is \$30 and the price of trees in the third class is \$50, which class should be completely harvested to attain the optimal sustainable yield? What is the optimal yield if there are 1000 trees in the forest?

(10 marks)

c) The federal government desires to inoculate its citizens against a certain flu virus. The virus has two strains, and the proportions in which the two strains occur in the virus population is not known. Two vaccines have been developed and each citizen is given only one of them. Vaccine 1 is 85% effective against strain 1 and 70% effective against strain 2. Vaccine 2 is 60% effective against strain 1 and 90% effective against strain 2. What inoculation policy should the government adopt?

(6 marks)

QUESTION THREE (20 MARKS)

a) Consider the economy described in the table below

Input Required per Dollar Output

	Manufacturing	Agriculture	Utilities
Manufacturing	\$ 0.50	\$ 0.10	\$ 0.10
Agriculture	\$ 0.20	\$ 0.50	\$ 0.30
Utilities	\$ 0.10	\$ 0.30	\$ 0.40

Suppose that the open sector has a demand for \$ 7900 worth of manufacturing products, \$ 3950 worth of agricultural products, and \$ 1975 worth of utilities

i) Can the economy meet this demand? (6 marks)

ii) If so, find a vector **x** that will meet it exactly.

(6 marks)

b) Find the trigonometric polynomial of order 4 that is the least squares approximation to the function $f(t) = t^2$ over the interval [*0*, *T*]. (8 marks)

QUESTION FOUR (20 MARKS)

a) Consider the transition matrix $P = \begin{pmatrix} 0.4 & 0.5 \\ 0.6 & 0.5 \end{pmatrix}$

i) Calculate $X^{(n)}$ for $n = 1, 2, 3, 4, 5$ if $X^{(0)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

ii) State why P is regular and find its steady-state vector. (8 marks)

b) Suppose that a game has a payoff matrix $A = \begin{pmatrix} -4 & 6 & -4 & 1 \\ 5 & -7 & 3 & 8 \\ -8 & 0 & 6 & -2 \end{pmatrix}$

i) If players R and C use strategies $p = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$ and $q = \begin{pmatrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{pmatrix}$ respectively, what is the expected payoff of the game?

(4 marks)

ii) If player C keeps his strategy fixed as in part (a), what strategy should player R choose to maximize his expected payoff?

(4 marks)

iii) If player R keeps her strategy fixed as in part (a), what strategy should player C choose to minimize the expected payoff to player R?

(4 marks)

QUESTION FIVE (20 MARKS)

a) Find the equation, center and radius of the sphere that passes through the four points $(0, 3, 2), (1, -1, 1), (2, 1, 0)$ and $(5, 1, 3)$

(7 marks)

b) A certain family consists of a mother, father, daughter, and two sons. The family members have influence, or power, over each other in the following ways: the mother can influence the daughter and the oldest son; the father can influence the two sons; the daughter can influence the father; the oldest son can influence the youngest son; and the youngest son can influence the mother. By the use of a directed graph model this family influence pattern and construct the vertex matrix

(5 marks)

c) An automobile mechanic (M) and a body shop (B) use each other's services. For each \$ 1.00 of business that M does, it uses \$ 0.50 of its own services and \$ 0.25 of B's services, and for each \$ 1.00 of business that B does, it uses \$ 0.10 of its own services and \$ 0.25 of M's services

i) Construct a consumption matrix for this economy

ii) How much must M and B each produce to provide customers with \$ 7000 worth of mechanical work and \$ 14000 (8 marks)