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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
FOURTH YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

KMA 2409 DESIGN AND ANALYSIS OF EXPERIMENTS

Date: 15TH AUGUST, 2024
Time: 8:30 AM – 11:30 AM

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

a) Briefly explain the importance of the three principles of design and analysis of experiments. **(6 Marks)**

b) In a randomized block design with 4 blocks and 6 treatments, the following is an incomplete ANOVA table for the experiment.

Source	df	SS	MSS	F
Treatments		35151.38		
Blocks				
Error			2180.942	
Total		68247.63		

Find the;

i. Degrees of freedoms of freedom. **(2 Marks)**
ii. Missing Sum of Squares. **(2 Marks)**
iii. Missing Mean Sum of Squares and F values. **(2 Marks)**
iv. Test for equality of treatment effects at 5% level of significance. **(2 Marks)**

c) An experiment was conducted to assess whether physical exercise alleviate depression, as follows; Some equivalently depressed people were taken. Each person was randomly allocated to one of the three groups: no exercise, 20 minutes of jogging per day or 60 minutes of jogging per day. At the end of a month, each participant was asked to rate how depressed they felt, on a Likert scale that runs from 1 ("totally miserable") through to 100 ("ecstatically happy"). The following data were recorded:

Group	Rating on depression scale						
No exercise 23	26	51	49	58	37	29	
Jogging for 20min 22	27	39	29	46	48	49	65
Jogging for 60min 59	66	38	49	56	60		

i. State the null and alternative hypothesis to be tested for this investigation. **(2 Marks)**
ii. Do the depression rates in the three groups differ at 5% level of significance? **(5 Marks)**

d) In a factorial experiment, there are two factors A and B, each having two levels, $(a_0,\ a_1)$ and $(b_0,\ b_1)$ respectively. Determine the possible treatment combination, design and X-matrices. **(5 marks)**

e) When are two Latin squares said to be orthogonal? Form orthogonal Latin Squares of size $s = 5$. **(4 Marks)**

QUESTION TWO: (20 MARKS)

a) Explain **TWO** reasons as to why you would prefer a Randomized Block Designs over a Completely Randomized Designs when designing an experiment. **(4 Marks)**

b) Four different tractors are to be considered in cultivating of a particular plot. It is decided that six different operators are to be used in a randomized block experiment to compare the tractors. The tractors are assigned at random to each operator. The following times in seconds, were recorded for cultivating a third of the given land:

Tractor	**Operator**						**Total**
	1	**2**	**3**	**4**	**5**	**6**	
1	425	393	396	399	429	436	**2478**
2	398	401	405	423	425	431	**2483**
3	402	405	413	434	449	451	**2554**
4	413	422	435	442	459	423	**2594**
Total	**1638**	**1621**	**1649**	**1698**	**1762**	**1741**	**10109**

i. Write the model of analysis and state what each symbol stands for. **(3 Marks)**

ii. Estimate the parameters of the experimental design. **(5 Marks)**

iii. Obtain the sum of squares and the Analysis of Variance (ANOVA) table. **(5 Marks)**

iv. Test the hypothesis at $\alpha = 0.05$ level of significance, that the operators perform at the same mean rate of speed. **(3 Marks)**

QUESTION THREE: (20 MARKS)

An engineer is interested in the effects of cutting speed (A), tool geometry (B), and cutting angle (C) on the life (in hours) of a machine tool. Two levels of each factor are chosen, and three replicates of a 2^3 design are run. The results are summarized below:

Treatment	**Replications (Blocks)**			**Total**
	I	**II**	**III**	
1	22	31	25	**78**
a	32	43	29	**104**
b	35	34	50	**119**
ab	55	47	46	**148**
c	44	45	38	**127**
ac	40	37	36	**113**
bc	60	50	54	**164**
abc	39	41	47	**127**
	327	**328**	**325**	**980**

a) Construct the design matrix and x-matrix. **(4 Marks)**

b) Estimate the main and interaction effects. **(7 Marks)**

c) Obtain the sum of squares and construct the ANOVA table. **(6 Marks)**

d) Which treatment combination are significant? Take $\alpha = 0.05$. **(3 Marks)**

QUESTION FOUR: (20 MARKS)

a) Consider a Latin square design with mode $y_{ij(k)} = \mu + \propto_i + \beta_j + T_k + \epsilon_{ij(k)}$ (symbols have the usual meaning) $i = 1,2,\dots,s;\ \ j = 1,2,\dots,s\ \ k = 1,2,\dots,s$. Suppose that the observation in k^{th} treament and i^{th} row belonging and j^{th} column is missing. Derive the estimator of the missing observation. **(8 Marks)**

b) Consider the a Latin square design with four treatments A, B, C and D shown below, with observation belonging to treatment A in the 1st row and the 1st column missing.

Yields		**Column**			
		I	**II**	**III**	**IV**
Rows	I	A(x)	B(37.3)	C(56.2)	D(52.1)
	II	D(50.1)	A(34.1)	B(29.3)	C(52.8)
	III	C(56.2)	D(51.9)	A(44.1)	B(33.2)
	IV	B(31.7)	C(62.1)	D(48.0)	A(45.7)

i. Evaluate the missing observation. **(4 Marks)**
ii. Test whether the treatments have the same effect at 1% level of significance. **(8 Marks)**

QUESTION FIVE: (20 MARKS)

a) Explain the reason as to why Balanced Incomplete Block Design (BIBD) is considered to be better than Randomized Block Design (RBD). **(2 Marks)**

b) An engineer is studying the mileage performance characteristics of 5 types of Gasoline additives. In the road tests he wishes to use cars as blocks, however because of time constraints, he must use an incomplete block design. He runs the balanced design with the 5 blocks as follows.

Additive	**Blocks (cars)**				
	1	2	3	4	5
1	-	17	14	13	12
2	14	14	-	13	10
3	12	-	13	12	9
4	13	11	11	12	-
5	11	12	10	-	8

Analyse the data from this experiment (use $\alpha = 0.05$) and draw conclusions.

(10 Marks)

c) Consider two mutually orthogonal Latin squares

L_1

A	B	C
B	C	A
C	A	B

L_2

$\propto$	β	γ
γ	$\propto$	β
β	γ	$\propto$

i. Using this Latin squares, obtain a BIBD with parameters
   a. $b = 12,\ v = 9,\ k = 3, r = 4, \lambda = 1$. **(5 Marks)**
ii. Complementary design to design obtained in (i). **(3 Marks)**