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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2022/2023 ACADEMIC YEAR
FOURTH YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

KMA 410 - MULTIVARIATE STATISTICAL METHODS I

Date:14th April 2022
Time:8.30am-10.301m

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Define
   i) A random variable. (1 mark)
   ii) Random Vector. (1 mark)
   iii) Random matrix. (1 mark)

b) Explain how a multivariate data are arranged. (3 marks)

c) The data below show the score of six students in mid-semester and end of semester examination

Mid-semester	28	12	17	24	30	9
End of semester	60	20	45	40	52	30

Estimate;
   i) Mean vector. (2 marks)
   ii) Covariance matrix. (4 marks)
   iii) Correlation Matrix. (2 marks)

d) Suppose that $\underline{X}' = (X_1, X_2, X_3)$ be a 3-dimensional random vector with a joint probability distribution

$$f(x_1, x_2, x_3) = \begin{cases} k(x_1x_2 + x_1x_3), & x_1 = 1, 2, 3 \;\; x_2 = 2,3,4 \;\&\; x_3 = 1,2 \\ 0, & elsewhere \end{cases}$$

Determine;
   i) Value of the constant K. (3 marks)
   ii) Marginal distribution of X_1. (3 marks)
   iii) Joint marginal distribution of X_2 and X_3. (2 marks)
   iv) Conditional distribution of X_1 given that $X_2 = x_2$ and $X_3 = x_3$. (2 marks)
   v) Are X_2 and X_3 jointly independent of X_1? Give a reason to your answer. (2 marks)

e) Given that a random vector $\underline{X}' = (X_1, X_2, X_3)$ has a mean vector $\underline{\mu}' = (10, 5, 30)$ and covariance matrix $\Sigma = \begin{bmatrix} 25 & 6 & 20 \\ 6 & 16 & 15 \\ 20 & 15 & 36 \end{bmatrix}$, find the mean and variance of $Y = X_1 + 3X_2 - X_3$.

(4 marks)

QUESTION TWO (20 MARKS)

a) Suppose that $\boldsymbol{X}_{p\times 1}$ be a p-dimensional random vector with mean $\boldsymbol{\mu}_{p\times 1}$ and covariance matrix covariance matrix $\boldsymbol{\Sigma}_{p\times p}$.

(i) Show that $\boldsymbol{\Sigma} = \boldsymbol{E}(\boldsymbol{X}-\boldsymbol{\mu})'(\boldsymbol{X}-\boldsymbol{\mu})$. (4 marks)

(ii) Suppose that $Y = \boldsymbol{a}'\boldsymbol{X}$ where **a** is a $1 \times p$ vector of constants. Show that the mean and variance of Y are $\boldsymbol{\mu_y} = \boldsymbol{a}'\boldsymbol{\mu}$ and $\boldsymbol{\Sigma_y} = \boldsymbol{a}'\boldsymbol{\Sigma a}$ respectively. (5 marks)

(iii) Suppose that $\boldsymbol{W} = \boldsymbol{AX}$ where **A** is a $q \times p$ matrix of constants. Show that the mean vector and covariance matrix of **W** are $\boldsymbol{\mu_w} = \boldsymbol{A\mu}$ and $\boldsymbol{\Sigma_w} = \boldsymbol{A\Sigma A}'$. (5 marks)

b) Given that a random vector $\boldsymbol{X}' = (X_1, X_2, X_3)$ has a mean vector $\boldsymbol{\mu}' = (-1, 0, 3)$ and covariance matrix $\boldsymbol{\Sigma} = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 9 & 3 \\ -1 & 3 & 4 \end{bmatrix}$. Let $\boldsymbol{Y} = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$ where $Y_1 = X_1 + X_2 - X_3$ and $Y_2 = 2X_2 + X_3$. Find the mean vector and variance covariance matrix of $\boldsymbol{Y}$. (5 marks)

QUESTION THREE (20 MARKS)

a) Write down the general form of a multivariate normal distribution. (2 marks)

b) Find the m.g.f. of the multivariate normal distribution in (a). (12 marks)

c) From the m.g.f. in (b), determine the mean and variance of a multivariate normal random variable. (6 marks)

QUESTION FOUR (20 MARKS)

a) If $\mathbf{X}^T = [X_1, X_2]$ is a bivariate distributed as $\mathbf{X} \sim N(\boldsymbol{\mu}_X, \Sigma_X)$ where $\boldsymbol{\mu}_X = \begin{bmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{bmatrix}$ and $\Sigma_X = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$. Show that the conditional distribution of $\mathbf{X}_1$ given $\mathbf{X}_2 = x_2$ is given as $N\left(\boldsymbol{\mu}_1 + \frac{\sigma_{12}}{\sigma_{22}}(\mathbf{x}_2 - \boldsymbol{\mu}_2), (1-\rho_{12}^2)\sigma_{11}\right)$ (10 marks)

b) The random vector $\mathbf{X}^T = [X_1, X_2, X_3, X_4]$ is normally distributed with a mean vector $\boldsymbol{\mu}_X{}^T = [2, 5, 3, 6]$ and covariance matrix

$$\Sigma_X = \begin{bmatrix} 11 & 5 & 2 & 3 \\ 5 & 4 & 1 & 0 \\ 2 & 1 & 3 & 0 \\ 3 & 0 & 0 & 2 \end{bmatrix}$$

Find the parameters of the distribution of conditional of X_1, X_3 given that $X_2 = 1$ and $X_4 = 4$.

(10 marks)

QUESTION FIVE (20 MARKS)

a) Suppose a coin is tossed three times and the side facing up is recorded. Let X_1 be the number of heads in the first two tosses and X_2 be the total number of tails in each outcome. Obtain;

i) The joint probability distribution of X_1 and X_2. (4 marks)

ii) Marginal distributions of X_1 and X_2. (2 marks)

iii) Mean vector of $\boldsymbol{X}' = (X_1, X_2)$. (3 marks)

iv) Covariance matrix of $\boldsymbol{X}$. (5 marks)

b) Let $\boldsymbol{X}$ be a four dimensional random vector with the joint pdf given by

$$f(x_1, x_2, x_3\ x_4) = \begin{cases} \frac{3}{4}(x_1^2 + x_2^2 + x_3^2 + x_4^2), & 0 < x_1, x_2, x_3, x_4 < 1 \\ 0, & Otherwise \end{cases}$$

Find the $P\left(X_1 < \frac{1}{2}, X_2 < \frac{3}{4}, X_4 > \frac{1}{2}\right)$ (6 marks)