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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2022/2023 ACADEMIC YEAR
THIRD YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS AND COMPUTER SCIENCE)
SPECIAL EXAMINATION

Date: 3rd August, 2022
Time: 11.30am –1.30pm

KMA 311 - PARTIAL DIFFERENTIAL EQUATIONS

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) Define a real valued function in more than one independent variable. (4marks)

b) State the domain and range of the functions below; (6marks)

Function	Domain	Range
$z=\sqrt{y-x^2}$		
$z=\sin xy$		
$w=\dfrac{1}{x^2+y^2+z^2}$		

c) Let $f(x,y)=x^2y+3x^2y^2-5$. Find

i. $\dfrac{\partial f}{\partial x}$

ii. $\dfrac{\partial^2 f}{\partial y^2}$ (4marks)

d) Describe the level surface of the function $f(x,y,z)=\sqrt{x^2+y^2+z^2}$ (5marks)

e) The plane $x=1$ intersects the paraboloid $z=x^2+y^2$ in a parabola. Find the slope of the tangent to the paraboloid at $(1,2,5)$. (3marks)

f) Given the following partial differential equations, state their linearity, order, dependent and independent variables;

i. $x^2\dfrac{\partial^3 R}{\partial y^3}=y^3\dfrac{\partial^2 R}{\partial x^2}$ (4marks)

ii. $\left(\frac{\partial z}{\partial u}\right)^2+\left(\frac{\partial z}{\partial v}\right)^2=1$ (4marks)

QUESTION TWO (20MARKS)

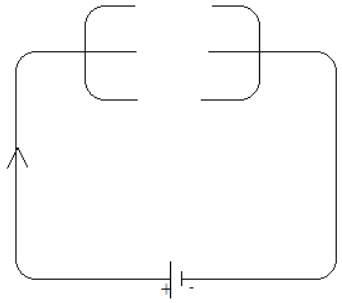
a) Let $f(x,y)=x^2+3xy+y-1$. Find

i) $\frac{\partial f}{\partial x}$

ii) $\frac{\partial f}{\partial y}$ at the point (4,-5) (4marks)

b) The function $yz-Inz=x+y$ defines z a function of the two independent variables x and y and given that the partial derivative exits, find $\frac{\partial z}{\partial x}$. (4marks)

c) The resistors of R_1, R_2 and R_3 Ohms are connected in parallel to make an R-Ohm resistor, where the value of R can be found from the equation $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$.



Find the value of $\frac{\partial R}{\partial R_2}$ when $R_1=30$, $R_2=45$ and $R_3=90$ Ohms. (6marks)

d) i). State and describe the equation for Transverse Vibrations of a Beam. (3marks)
ii.Give the following characteristics of the equation; order, linearity and homogeneity. (3marks)

QUESTION THREE (20MARKS)

a) Show that $u(x,t)=e^{-8t}\sin 2x$ is a solution to the boundary-value problem $\frac{\partial u}{\partial t}=2\frac{\partial^2 u}{\partial x^2}$ given $u(0,t)=u(\pi,t)=0$ and $u(x,0)=\sin 2x$. (8marks)

b) Show that $V=F(y-3y)$ where F is an arbitrary function is a general solution of the partial differential equation $\frac{\partial v}{\partial x}+3\frac{\partial v}{\partial y}=0$. (5marks)

Hence find the particular solution which satisfies the equation $v(0,y)=4\sin y$ (4marks)

c) Solve the partial differential equation $t\frac{\partial^2 u}{\partial x \partial y}+2\frac{\partial u}{\partial x}=x^2$ (3marks)

QUESTION FOUR (20MARKS)

a) Solve the boundary-value problem $\frac{\partial u}{\partial x}=4\frac{\partial u}{\partial y}$, $u(0,y)=8e^{-3y}$ by the method of separation of variables. (10marks)

Further find $u(x,y)$ given that $u(0,y)=8e^{-3y}+4e^{-5y}$ (6marks)

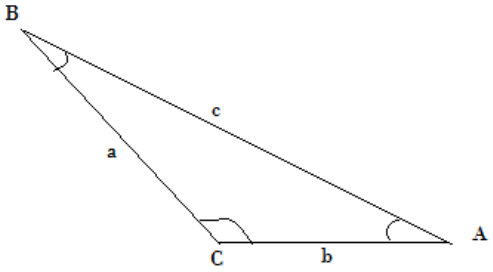
b) Find $\lim_{(x,y)\to(0,0)}\frac{x^2-xy}{\sqrt{x}-\sqrt{y}}$ (4marks)

QUESTION FIVE (20MARKS)

a) Find $\frac{\partial^2 f}{\partial y^2}$ if $f(x,y)=\frac{2y}{y+\cos x}$ (6marks)

b) Find f_{yxyz} if $f(x,y,z)=1-2xy^2z+x^2y$ (4marks)

c) Given the obtuse angle triangle ABC, express A implicitly as a function of a, b and c. Hence calculate $\frac{\partial A}{\partial a}$ (5marks)



d) Solve the boundary value problem $\frac{\partial u}{\partial t}=4\frac{\partial^2 u}{\partial x^2}$, $u(0,t)=0$, $u(3,t)=0$, $u(x,0)=10\sin 2\pi x-6\sin 4\pi x$ (5marks)