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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
FOURTH YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

KMA 2403 TIME SERIES ANALYSIS

Date: 12TH AUGUST, 2024
Time: 8:30 AM – 10:30 AM

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

(a) Discuss four uses of time series analysis in statistics. **(4 Marks)**

(b) Consider the AR(2) process and show that $X_t = 0.8X_{t-1} - 0.15X_{t-2} + e_t$ is weakly stationary **(6 Marks)**

(c) Given the following observation of a time series for $n = 10$

t	1	2	3	4	5	6	7	8	9	10
X_t	47	64	23	71	38	64	55	41	59	48

Find

i. Sample auto-covariance $r(1)$ and $r(2)$ **(5 Marks)**

ii. Sample auto-correlation $p(1)$ and $p(2)$ **(5 Marks)**

(d) Consider MA (2) process given by $Y_t = e_t - \frac{5}{10}e_{t-1} - \frac{4}{10}e_{t-2}$

Determine

i. Whether the process is invertible **(3 Marks)**

ii. The covariance generating function **(4 Marks)**

iii. The autocorrelation function **(3 Marks)**

QUESTION TWO: (20 MARKS)

a) Consider a set of independent and identically distributed random variable $\{e_t\}$ such that E (e_t) is zero and variance of e_t is $\sigma_e{}^2$. Let the process be given by $X_t = \emptyset e_{t-1} + e_t$ where $\emptyset$ is a constant.

i) Find the $E(x_t x_{t+m})$ **(5 Marks)**

ii) Show that X_t is weakly stationary. **(5 Marks)**

b) Given $X_t = \theta X_{t-1} + e_t$

i. Find the spectral density function of an AR (1) process **(5 Marks)**

ii. Show that the spectral density function of an AR (1) process is given by **(5 Marks)**

$$f(\omega) = \frac{\sigma^2}{2\pi(1-2\theta cos\omega+\sigma^2)}$$

NB: The standard spectral density function of the white noise if $f_e\,(\omega) = \frac{\sigma^2}{2\pi}$

QUESTION THREE: (20 MARKS)

a) Discuss four components of a time series data. **(8 Marks)**

b) Fit a local polynomial of degree 3 to 7 consecutive data points given the weight.

$$w = \frac{1}{21}(-2,\ \ 3,\ \ 6,\ \ 7,\ \ 6,\ \ 3,\ \ -2)$$

i. Find derivatives with respect to parameters **(5 Marks)**

ii. Fit a local polynomial and extract the coefficients **(7 Marks)**

QUESTION FOUR: (20 MARKS)

a) Determine whether the process is invertible **(4 Marks)**

$X_t = e_t + 0.7e_{t-1} - 0.2e_{t-2}$

b) Determine if the process $x_t = 1.5x_{t-1} - 0.5x_{t-2} + e_t$ is stationary **(4 Marks)**

c) Find the covariance generating function of the MA (2) process given by

$x_t = \frac{1}{3}e_{t-1} + \frac{1}{3}e_t + \frac{1}{3}e_{t+1}$ **(6 Marks)**

Hence the autocorrelation function **(6 Marks)**

QUESTION FIVE: (20 MARKS)

a) Consider the AR (2) process given by $x_t = \frac{4}{5}x_{t-1} - \frac{15}{100}x_{t-2} + e_t$.Show that x_t

i. Is stationary **(5 Marks)**

ii. Find its autocorrelation function. **(5 Marks)**

b) Given the following observation of a time series for n = 10

t	1	2	3	4	5	6	7	8	9	10
Xt	0.812	1.657	2.537	3.431	4.329	5.254	6.174	7.104	8.044	8.956

Find;

i. Sample auto-covariance $r(1)$ and $r(2)$ **(5 Marks)**

ii. Sample auto-correlation $p(1)$ and $p(2)$ **(5 Marks)**