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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
SECOND YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)
(SPECIAL EXAMINATION)

KMA 209: ALGEBRA

DATE: 6TH DECEMBER 2024

TIME: 2:30PM – 4:30PM

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

- 1) Define the following terms
 - (i) Group
 - (ii) Binary operation
 - (iii) Permutation **(6 Marks)**
- b) Prove that for all $a, b \in G$ then $(ab)^{-1} = b^{-1}a^{-1}$ **(4 Marks)**
- c) Define $*$ on Q^+ by $a * b = \frac{ab}{2}$. Show that $(Q^+, *)$ is a group. **(4 Marks)**
- d) Define an abelian group and prove that every cyclic subgroup is abelian. **(4 Marks)**
- e) Show that every division ring is a ring without zero divisor. **(5 Marks)**
- f) Define transposition and list the even and odd permutations in S_3 **(4 Marks)**
- g) Prove that every field is an integral domain. **(3 Marks)**

QUESTION TWO: (20 MARKS)

- 1) Let G denote the set of all ordered pairs of real numbers with non-zero first component of the binary operation $*$ is defined by $(a, b) * (c, d) = (ac, bc + d)$. Show that $(G, *)$ is a non-abelian group. **(8 Marks)**
- 2) Let A be a non-empty set and let S_A be the collection of all permutations of A . Show that S_A is a group under permutation multiplication. **(7 Marks)**
- 3) An identity element (if it exist) of mathematical system $(S, *)$ is unique. Prove. **(5 Marks)**

QUESTION THREE: (20 MARKS)

- a) Let m be a fixed positive integer in Z . Define the relation $\equiv_n$ on Z as follows for all

$$x, y \in Z. \ x \equiv_n y \text{ iff } \frac{n}{x-y} \text{ i.e } x-y = nk. \text{ Show that } \equiv_n \text{ is an equivalence relation in } Z.$$

(6 Marks)

- b) Let $f : G \rightarrow G_1$ be a group homomorphism. Show that kernel of f is a normal subgroup of G . **(8 Marks)**

- c) Show that every subgroup of an abelian group is normal (6 Marks)

QUESTION FOUR: (20 MARKS)

- a) Define normal subgroup and prove that every subgroup of index 2 is normal. (6 Marks)

- b) Let H be a normal subgroup of G . Denote the set of all left cosets $\{aH \mid a \in G\}$ by $\frac{G}{H}$ and

define $*$ in $\frac{G}{H}$ for all $aH, bH \in \frac{G}{H}$ by $(aH)*(bH) = abH$. Show $\left(\frac{G}{H}, *\right)$ is a group

(8 Marks)

- c) Let R_1 and R_2 be subrings of R . Show that $R_1 \cap R_2$ is a subring of R . (6 Marks)

QUESTION FIVE: (20 MARKS)

- a) State the Lagrange's Theorem (3 Marks)

- b) List all the elements of a symmetric group of order 3 and construct a multiplication table for S_3 (12 Marks)

- c) Prove that any two, right and left cosets of H in G are disjoint. (5 Marks)