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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
FIRST YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE (COMPUTER SCIENCE)

KMA 2106: PROBABILITY AND STATISTICS I

DATE: 6TH DECEMBER 2024

TIME: 8:30AM – 10:30AM

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

- a) A random variable X has a probability distribution given by

$$f(x) = \begin{cases} k(1-x), & 0 \leq x \leq 1 \\ 0, & \text{Otherwise} \end{cases}$$

- i) Find the value of the constant k. **(3 Marks)**

- ii) Evaluate the $P\left(X \leq \frac{1}{3}\right)$. **(2 Marks)**

- b) For a random variable X with p.m.f given by

x	0	1	2	3	4	5
$P(X = x)$	0.1681	0.3601	0.3087	0.1323	0.0284	0.0024

determine the;

- i) mode. **(2 Marks)**

- ii) Upper and Lower quartiles. **(3 Marks)**

- c) A random variable X has a pdf given by

$$f(x) = \begin{cases} 3e^{-3x}, & x > 0 \\ 0, & \text{Otherwise} \end{cases}$$

- I) Find the moment generating function of X. **(3 Marks)**

- II) Use the moment generating function obtained in i) to find

- i) Mean of X. **(2 Marks)**

- ii) Variance of X. **(3 Marks)**

- d) On average, one out of 10 telephones numbers are found to be busy. Six telephone numbers are selected at random. Let X be the number of busy telephone numbers out of the six selected ones.

- i) Write down the probability distribution of X. **(1 Marks)**

- ii) Find the probability that at most two of them will be busy. **(3 Marks)**

- e) The final exam scores in a statistics class were normally distributed with a mean of 63 and a standard deviation of 5. Find the probability that a randomly selected student score between 60 and 70. **(3 Marks)**
- f) A construction firm has placed an order that they require a consignment of wires which have a mean length of 10.5 meters with a standard deviation of 1.7 m. The company which produces the wires delivered 90 wires, which had a mean length of 9.2 m. The construction company rejected the consignment on the grounds that they were different from the order placed. Conduct a statistical test to indicate whether you support or not support the action taken by the construction company at 5% level of significance. **(5 Marks)**

QUESTION TWO: (20 MARKS)

- a) Let X be a continuous random variable with a geometric distribution given by

$$f(x) = \begin{cases} \theta e^{-\theta x}, & x > 0 \\ 0, & \text{Otherwise} \end{cases}$$

Without using moment generating function, show that

- i) $E(X) = \frac{1}{\theta}$. **(4 Marks)**
- ii) $Var(X) = \frac{1}{\theta^2}$. **(6 Marks)**
- b) The length of time a computer battery lasts from the time it is manufactured is known to be exponentially distributed with a mean life of 10 years. What is the probability that the battery last for;
- i) Less than 5 years. **(3 Marks)**
- ii) Between 7 and 12 years. **(3 Marks)**
- iii) 3 more years given that it has lasted for 4 years. **(4 Marks)**

QUESTION THREE: (20 MARKS)

- a) Consider a binomial random variable X with probability distribution function given by

$$f(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x = 0, 1, \dots, n \\ 0, & \text{elsewhere} \end{cases}$$

- I) Determine the moment generating function of X . **(4 Marks)**
- II) Use the mgf above to obtain;
- i) the mean. **(2 Marks)**
- ii) the variance. **(3 Marks)**
- b) Suppose 40% of the population approves of the job the governor is doing, and that 20 individuals are drawn at random from the population. What is the probability that;
- i) exactly 7 people will support the governor? **(2 Marks)**
- ii) fewer than 5 people will support the governor? **(3 Marks)**
- c) The number of tasks sent to a company printer in busy time is known to be Poisson distributed with parameter $\lambda = 2$ per minute. Compute the probability that;
- i) At least 2 tasks are sent in a minute. **(3 Marks)**
- ii) Not more than three tasks are sent in three minutes. **(3 Marks)**

QUESTION FOUR: (20 MARKS)

- a) Medical researchers have found out that the length of human pregnancy is normally distributed with mean of 40 weeks and a standard deviation of 2 weeks.
- I) Of 500 pregnant women, how many would you expect their pregnancy to last for;
- i) Less than 37 weeks? **(3 Marks)**
- ii) Between 38 and 41 weeks? **(3 Marks)**
- II) If 90% of women have pregnancies not lasting for more than K weeks, find the value of K. **(3 Marks)**
- b) The amount of time, in minutes, that a person must wait for a bus is uniformly distributed between 0 and 15 minutes, inclusive. What is the probability that;
- i) a person waits fewer than 12.5 minutes? **(2 Marks)**
- ii) the waiting time will be between 0.5 standard deviations from the mean. **(4 Marks)**
- c) A school has 20 students in a class, 8 of whom are girls and 12 are boys. The teacher wants to select a group of 5 students to participate in a special project. What is the probability that;
- i) exactly 3 girls will be chosen in this group of 5? **(2 Marks)**
- ii) at least 2 girls will be chosen in this group of 5? **(3 Marks)**

QUESTION FIVE: (20 MARKS)

- a) Highlight the steps in hypothesis testing. **(6 Marks)**
- b) A simple random sample of 10 people from a certain population has a mean age of 27. Can we conclude that the mean age of the population is less than 30? The variance is known to be 20. Let $\alpha = .05$. **(5 Marks)**
- c) The average cost (in hundreds of shillings) of a hotel room in Nairobi is said to be 168 per night. To determine if this is true, a random sample of 25 hotels is taken and resulted in $\bar{x} = 172.5$ and $s = 15.40$.
- i) Test the appropriate hypotheses at $\alpha = 0.05$. **(5 Marks)**
- ii) Compute 95% confidence intervals of the average cost of hotel rooms in Nairobi. **(4 Marks)**