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KIRIRI WOMEN'S UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION FOR 2024/2025 ACADEMIC YEAR
THIRD YEAR, SECOND SEMESTER EXAMINATION
(SPECIAL EXAMINATION)

KCS 310: REAL ANALYSIS

DATE: 11TH DECEMBER,2024
TIME: 11:30AM-1:30PM

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

a) Give an example of a set which has got an infimum, a minimum and a supremum but does not have a maximum element. **(2 Marks)**

b) State whether the following sets are open, closed or neither

i) $S = \{1,2,3,4\}$ **(1 Mark)**
ii) $S = (2, \infty)$ **(1 Mark)**
iii) $\mathbb{R}$ **(1 Mark)**

c) Let (X, ρ) be a metric space. Define the following terms:

i) Neighborhood of a point. **(1 Mark)**
ii) Interior point. **(2 Marks)**
iii) Limit point. **(2 Marks)**
iv) Isolated point. **(1 Mark)**

d) Given a set $S = \{1,3\} \cup (4,11)$. Find the following:

i) S° **(1 Mark)**
ii) $\bar{S}$ **(1 Mark)**
iii) Is S **(1 Mark)**
iv) ∂S **(1 Mark)**

e) Prove that the set of irrational numbers is uncountable. **(3 Marks)**

f) Prove that an empty set is open. **(4 Marks)**

g) Show that the infinite series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent. **(2 Marks)**

h) Prove using mathematical induction that for all $n \geq 1$, $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ **(3 Marks)**

i). Let X be a non-void set and $\rho : \boldsymbol{X} \times \boldsymbol{X} \rightarrow \boldsymbol{X}$ be defined by $\rho(x, y) = |x - y|$ Show that (X, ρ) is a metric space. **(3 Marks)**

QUESTION TWO: (20 MARKS)

a) Prove that the intersection of an arbitrary family of closed sets is closed. **(6 Marks)**

b) Prove that the set of rational numbers is countable. **(6 Marks)**

c) i) Show that the infinite set $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots \ldots \ldots \ldots \ldots \ldots . is\ bounded$ **(2 Marks)**

ii) Determine the supremum and the infimum of the set in question c i) above **(2 Marks)**

d) Use ratio test to test the convergence of the series $\sum_{n=1}^{\infty} \frac{n!(n+1)!}{(3n)!}$. **(4 Marks)**

QUESTION THREE: (20 MARKS)

a) Prove that every convergent sequence has a unique limit. **(6 Marks)**

b) Give a counter example to show that if sequence is bounded it is not necessarily convergent.

(4 Marks)

c) For any four numbers a, b, c, d assume that a<b and c<d and prove that a+c < b+d.

(5 Marks)

d) Prove that for positive numbers x and y , x<y then if and only if $x^2 < y^2$.

(5 Marks)

QUESTION FOUR: (20 MARKS)

a) Let X be a non-void set and $\rho : \boldsymbol{X} \times \boldsymbol{X} \rightarrow \boldsymbol{X}$ be defined by

$\rho(x, y) = \begin{cases} 1\ if\ \ x \neq y \\ 0\ \ if\ \ x = y. \end{cases}$ Show that $(X, \ \rho)$ is a metric space. **(6 Marks)**

b) Prove that is a set S has a minima in $\mathbb{R}$, then this minima is unique. **(4 Marks)**

c) Given the set $S = [-\infty, 4) \cup \{5, 9\} \cup [6,7)$, find:

i. A° **(1 Mark)**

ii. ∂A **(1 Mark)**

iii. $(A^c)^{\circ}$ **(2 Marks)**

iv. $\overline{A}$ **(1 Mark)**

d) State and prove the principle of Archimedean. **(5 Marks)**

QUESTION FIVE:(20 MARKS)

a) State Cauchy root test and hence use it to test the convergence of

$$\sum_{n=0}^{\infty} \left(\frac{5n-3n^3}{7n^3+2}\right)^n$$ **(5 Marks)**

b) Use integral test to test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n-1}}$ **(5 Marks)**

c) i) state the completeness axiom. **(1 Mark)**

ii) Illustrate using an example that the set of rational numbers doesn't satisfy the completeness axiom. **(4 Marks)**

d) Prove that the intersection of finite number of open sets is open. **(5 Marks)**