



Kasarani Campus
Off Thika Road
Tel. 2042692 / 3
P. O. Box 49274, 00100
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2016/2017 ACADEMIC YEAR
SECOND YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS)

Date: 12th August, 2016.
Time: 8.30am – 10.30am

KMA 205 - BASIC NUMBER THEORY

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

- a) If a/b and c/d show that $a/b = c/d$. (5 Marks)
- b) Show that $\sqrt{11}$ is irrational. (5 Marks)
- c) The square of an odd integer is odd and the square of an even integer is even proof. (6 Marks)
- d) Is 117 a prime number? (4 Marks)
- e) Show that $1 + 5 + 5^2 + \dots + 5^{11}$ is composite. (5 Marks)
- f) Show that if p is prime and the $GCD(a, p) > 1$, then p/a . (5 Marks)

QUESTION TWO (20 MARKS)

- a) Prove that every odd number is either of the form $4p + 1$ or $4p + 3$. $p \in \mathbb{Z}$. (5 Marks)
- b) Prove that for integer n , $n + 2$, $n + 4$ then one of them is divisible by 3. (5 Marks)
- c) Use Eratosthenes's sieve to find the prime numbers less than 90. (5 Marks)
- d) Show that we cannot have 3 consecutive odd numbers other than 3, 5, 7 such that they are all prime. (5 Marks)

QUESTION THREE (20 MARKS)

- a) Show that whenever d is the GCD of a and b (a, b) then $-d$ is also the GCD of a and b . (5 Marks)
- b) Show that there are only 2 GCDs for any two pairs a and b . (5 Marks)
- c) Express the GCD of $(128, 30)$ in the form $n(128) + m(30)$. (5 Marks)
- d) Prove that if $a, b, c, d \in \mathbb{Z}$ and n is a positive integer and $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$ then $a + c \equiv b + d \pmod{n}$. (5 Marks)

QUESTION FOUR (20 MARKS)

- a) State Fermat's little theorem. (2 Marks)
- b) Use Fermat's little theorem to obtain $3^{60} \pmod{61}$. (5 Marks)
- c) By considering $f(x, y) \pmod{4}$ show that $f(x, y) = y^2 - x^2 - 2 = 0$ has no solution. (6 Marks)
- d) Express $\sqrt{7}$ as continued fraction. (7 Marks)

QUESTION FIVE (20 MARKS)

- a) State the Chinese remainder Theorem. (2 Marks)
- b) If a / b and c / d is it true that $a + c / b + d$. (5 Marks)
- c) Find $\varphi(60)$ (3 Marks)
- d) Find the set of solution z to the congruence $3z + 4 = 6 \pmod{7}$. (5 Marks)
- e) Find the GCD of the two numbers $(37129, 14659)$ using Euclidean algorithm. (5 Marks)