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KIRIRI WOMEN'S UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2024/2025 ACADEMIC YEAR
FIRST YEAR, SECOND SEMESTER EXAMINATION
FOR THE BACHELOR OF BUSINESS AND INFORMATION TECHNOLOGY
KBI 2108 – DISCRETE STRUCTURES

Date: 13TH December 2024
Time: 830AM – 10:30AM

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS
QUESTION ONE (30 MARKS)

a) Determine how each of the following pairs of sequences of bits is processed by an *AND* gate:
   i.   110001, 101101   (2 Marks)
   ii. 10001111, 00111100   (2 Marks)

b) Let p be "Audrey speaks French" and let q be "Audrey speaks Danish". Give a simple verbal sentence which describes each of the following:
   i. $\neg\neg p$ (1 Mark)
   ii. $\neg(\neg p \wedge \neg q)$ (1 Mark)

c) Verify that the proposition $p \vee \neg(p \wedge q)$ is a tautology   (4 Marks)

d) Using the following sets: $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{4, 5, 6, 7, 8, 9\}$,
      $C = \{1, 3, 5, 7, 9\}$, $D = \{2, 3, 5, 7, 8\}$, find:
   i. $A \oplus B$ (3 Marks)
   ii. $C \oplus D$ (3 Marks)

e) A graph G has 21 edges, 3 vertices of degree 4 and the other vertices are of degree 2. Find the number of vertices in G (5 Marks)

f) Find the sum-of- products expansion for the function $F(x, y, z) = (x + y)\,\bar{z}$ (5 Marks)

g) Prove the following proposition $p(n): 1 + 4 + 7 + \cdots + (3n - 2) = \dfrac{n(3n-1)}{2}$

                                                                                     (4 Marks)

QUESTION TWO (20 MARKS)

a) Given the sequence of bits $A = 1100110110$, $B = 1110000111$, and
       $C = 1010010110$, find
   i. $C(\overline{A} + B)$ (3 Marks)
   ii. $A\,\overline{(B + C)}$ (3 Marks)

b) Verify that the proposition $(p \wedge q) \wedge \neg(p \vee q)$ is a contradiction   (5 Marks)

c) Define the following terms in relation to graphs:
   i.   Isomorphism of graphs   (2 Marks)
   ii. A subgraph   (2 Marks)
   iii. Induced subgraph   (2 Marks)

d) Let $N = \{0, 1, 2, 3, \dots\}$. Define $f: N \to N$ as $f(m) = 3^m$. Show that f is a monoid homomorphism from $(N, +) \to (N, .)$ where $(N, +)$, $(N, .)$ are monoids under usual addition and multiplication respectively   (3 Marks)

QUESTION THREE (20 MARKS)

a) Prove the following proposition:
$$P(n): 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$
(5 Marks)

b) By use of a Venn diagram denote the following sets
   i. $A^C \cap (B \setminus C)$ (2 Marks)
   ii. $A \cap B \cap C^C$ (2 Marks)

c) Find x, y, and z if $(2x, \ x + y, \ x - y - 2z) = (4, \ -1, \ 3)$ (4 Marks)

d) A farmer buys three cows, two pigs, and four hens from a man who has six cows, five pigs, and eight hens. How many choices does the farmer have (4 Marks)

e) Define the terms: Converse, Inverse, and Contrapositive (3 Marks)

QUESTION FOUR (20 MARKS)

a) Solve the recurrence relation $a_{n+2} = a_{n+1} + a_n, \ n \geq 0, \ a_0 = 0, \ a_1 = 1$ (8 Marks)

b) Given $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$. Let R be the following relation from A to B:
$$R = \{(1, y), (1, z), (3, y), (4, x), (4, z)\}$$
   i. Determine the matrix of the relation (1 Mark)
   ii. Draw the arrow diagram of R (2 Marks)
   iii. Find the inverse relation R^{-1} of R (1 Mark)
   iv. Determine the domain and range of R (2 Marks)

c) Determine the contrapositive of each of the following statements:
   i. If Erick is a poet, then he is poor. (1 Mark)
   ii. Only if Mark studies will he pass the test (1 Mark)

d) Let $A = \{a, b, c, d, e, f, g, h\}, \ B = \{1, 2, 3, 4, 5\}$. How many elements are there in $P(A \times B)$, the power set of $A \times B$ (4 Marks)

QUESTION FIVE (20 MARKS)

a) Using the truth table verify the De Morgan`s law for logic $\neg(p \vee q) \equiv \neg p \wedge \neg q$
(2 Marks)

b) Find the value of $1.0 + \overline{(0 + 1)}$. (3 Marks)

c) A class contains 10 students with 6 men and 4 women. Find the number n of ways to:
   i. Select a $4-$ member committee from the students (2 Marks)
   ii. Select a $4-$ member committee with 2 men and 2 women (2 Marks)
   iii. Elect a president, vice president, and treasurer (2 Marks)

d) Show that the negation of $p \rightarrow q$ is logically equivalent to $p \wedge \neg q$ (2 Marks)

e) Given $A = \{1, 2\}, \ B = \{x, y, z\}$, and $C = \{3, 4\}$. Find: $A \times B \times C$. (4 Marks)

f) Write the negation of each of the following statements as simple as possible:
   i. If she works, she will earn money (1 Mark)
   ii. He swims if and only if the water is warm (1 Mark)
   iii. If it snows, then they do not drive the car (1 Mark)