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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2022/2023 ACADEMIC YEAR
THIRD YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS AND COMPUTER SCIENCE)

Date: 12th April, 2022
Time: 8.30am – 10.30am

KMA 302 - COMPLEX ANALYSIS 1

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Express the complex numbers in polar form;

i) $z = -\sqrt{6} - \sqrt{2}i$ (2 Marks)

ii) $z = -5 + 5i$ (2 Marks)

b) Evaluate $(1+i)^6 + (1-i)^3$ (4 Marks)

c) Simplify $\dfrac{(\cos 5\theta + i\sin 5\theta)(\cos 4\phi - i\sin 4\phi)}{(\cos 2\theta - i\sin 2\theta)(\cos 3\phi + i\sin 3\phi)}$ (4 Marks)

d) Apply the Cauchy residue theorem to evaluate $\int_C \frac{2z-1}{z(z+1)(z-3)} dz$ where C is the circle $|z| = 2$ (5 Marks)

e) Use De Moivre's theorem to find the 5th power of the complex number $z = 2(\cos 24^\circ + i\sin 24^\circ)$ (3 Marks)

f) Find the complex cube roots of $8(\cos 60^\circ + i\sin 60^\circ)$ (3 Marks)

g) Evaluate the following integral $\int_c \dfrac{1}{z(z-1)(2z-1)} dz \; c : |z| = 2$ using Cauchy's theorem (5 Marks)

h) Show that $f(z) = \frac{3z^4 - 2z^3 + 8z^2 - 2z + 5}{z - i}$ is not continuous at $z = i$. (2 Marks)

QUESTION TWO (20 MARKS)

a) Show that $\sin 5\theta = 16\, sin^5\theta - 20 sin^3\theta + 5\sin\theta$ (6 Marks)

b) Given that $z_1 = 2\sqrt{3}\, i + \ 2$ and $z_2 = -3i$; use polar coordinates to evaluate

i) $z_1 . z_2$ (3 Marks)

ii) $\frac{z_1}{z_2}$ (3 Marks)

c) If $(z)^{\frac{1}{3}} = a + ib$, where $z = x + iy,\ x, y, a, b \in \boldsymbol{R}$, show that $\frac{x}{a} - \frac{y}{b} = \mathbf{-2}(\boldsymbol{a^2} + \boldsymbol{b^2})$ (4 Marks)

d) Find the fourth roots of unity (4 Marks)

QUESTION THREE (20 MARKS)

a) Let $u(x, y) = 2x(1 - 2y)$

i) Show that $u(x, y)$ is harmonic (3 Marks)

ii) Find $v(x, y)$ such that *f(z)* is analytic (5 Marks)

iii) Express *f (z)* in terms of *z*. (2 Marks)

b) Evaluate $\int \bar{z}\, dz$ from $z = 0$ to $z = 4 + 2i$ along the line joining $z = 0$ to $z = 2i$ and the line joining $z = 2i$ to $z = 4 + 2i$.

(7 Marks)

c) Solve the equation $z^2 = \bar{z}$, where $= x + iy$ (3 Marks)

QUESTION FOUR (20 MARKS)

a) Explain the nature of the transformation $w = z^2$ considering the semi-circle with centre the origin and the radius *r* on the *z-plane.*

(6 Marks)

b) Find the bilinear transformation which maps the points $z_1 = 2,\ z_2 = i, z_3 = -2$ into the points $w_1 = 1, w_2 = i, w_3 = -1$ respectively.

(8 Marks)

c) Find the residues of $f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$

(6 Marks)

QUESTION FIVE (20 MARKS)

a) Expand $f(z) = \frac{z}{(z+1)(3+z)}$ in a Laurent series valid for;

i)$|z| < 1$ (2 Marks)

ii) $1 < |z| < 3$ (3 Marks)

b) Use Cauchy's Residue Theorem to evaluate the real integral

$\int_{-\alpha}^{\alpha} \frac{dx}{(x^2+1)(x^2+9)}$ (8 Marks)

c) Use Residue theorem to evaluate $\int_{0}^{2\pi} \frac{d\theta}{5-3\cos\theta}$ (7 Marks)