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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
FOURTH YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE(MATHEMATICS)

KMA 2418: QUALITY CONTROL METHODS AND ACCEPTANCE SAMPLING

DATE: 9TH DECEMBER 2024
TIME: 8:30AM – 10:30AM

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

a) A process is producing items with a nominal dimension of 50 mm. The specification limits are 49.5 mm and 50.5 mm. The process standard deviation is 0.15 mm, and the process mean is 50 mm. Calculate the process actual capability index C_{pk}. and comment whether the process is capable **(5 Marks)**

b) A company uses a single-sampling plan by attributes to inspect a lot of 500 items. The sampling plan is given by $n = 50$ and $c = 2$, where n is the sample size and c is the acceptance number. The historical defective rate for the production process is 4%.

i. What is the probability of accepting a lot with 4% defectives? **(4 Marks)**

ii. Calculate the AOQ (Average Outgoing Quality) assuming 100% inspection of rejected lots **(3 Marks)**

c) Consider quality control problem of Coca-Cola Company. The company inspector has taken 21 samples with 4 observations each of the volume of bottles filled as shown below.

Sample No.	1	2	3	4	5	6	7	8	9	10
$\overline{X}_i$	15.91	15.99	15.92	15.93	15.98	16.03	15.96	15.93	15.96	15.83
Range (R_i)	0.19	0.27	0.17	0.46	0.47	0.20	0.46	0.20	0.21	0.30

Cont.	11	12	13	14	15	16	17	18	19	20	21
	15.99	15.96	15.83	15.91	16.05	15.99	15.86	16.01	15.98	16.02	16.0
	0.29	0.43	0.24	0.37	0.31	0.29	0.33	0.34	0.28	0.20	0.23

Given that the standard deviation of bottling operations is 0.14 grams while the process mean is unknown,

i. Construct the control limits for this process **(3 Marks)**

ii. Plot the X-bar chart **(4 Marks)**

iii. Hence or otherwise, determine whether the process is in control or not **(2 Marks)**

d) Given the following sample variances from a different production process:

$$S^2 = \{1.20, 1.10, 1.40, 2.30, 1.15, 1.35, 1.60, 1.75, 1.25, 1.85\}$$

Obtain the control limits and hence comment on whether the process under control or not given that $X^2_{0.001,\,9} = 31.264$ **(5 Marks)**

e) A company uses a single-sampling plan for inspecting shipments of 500 products. The plan calls for inspecting 50 randomly selected items and rejecting the shipment if 4 or more defectives are found. The acceptable quality level (AQL) is 1%. Calculate the consumer's risk if the actual defect rate is 4%.

(4 Marks)

QUESTION TWO: (20 MARKS)

a) A factory produces 10,000 items every day, and over 20 days, the number of defective items was recorded as follows:

Day	1	2	3	4	5	6	7	8	9	10
Defective Items	120	150	100	130	160	110	140	130	150	110

Day	11	12	13	14	15	16	17	18	19	20
Defective Items	130	140	160	150	120	110	140	130	110	120

Use the data to calculate the following:

i. The proportion of defective items each day and the overall proportion of defective items. **(5 Marks)**

ii. Control limits for the p-chart and comment on the process stability **(4 Marks)**

b) You are monitoring the quality of bolts being produced in a factory. The following data shows the lengths (in mm) of bolts in 10 samples, each containing 4 bolts.

Sample	Bolt 1	Bolt 2	Bolt 3	Bolt 4
1	34	32	35	33
2	31	30	32	34
3	36	37	34	35
4	33	34	35	32
5	35	34	36	33

Sample	Bolt 1	Bolt 2	Bolt 3	Bolt 4
6	34	36	37	35
7	31	37	36	33
8	32	34	30	31
9	33	36	35	31
10	30	33	38	32

Given that $n = 4$ and using control chart constants $D_3 = 0$ and $D_4 = 2.282$,

i. Construct the control limits for the *R*-chart. **(5 Marks)**

ii. Comment on the process stability. **(2 Marks)**

c) The following data represents the sample variances S^2 of 10 samples, each of size 5, from a production process:

$$\{0.92, 1.15, 0.88, 1.07, 1.02, 1.25, 1.08, 0.97, 0.94, 1.12\}$$

Given that the target population variance $\sigma^2 = 1$ and using the following control chart constants:

$$B_3 = 0, \qquad B_4 = 2.114$$

Construct the S^2 control chart and determine whether the process is in control. **(4 Marks)**

QUESTION THREE: (20 MARKS)

a) Consider a resistor manufacturing company problem. A certain type of resistor is manufactured so that when operating properly, the value of $\mu = 5.1$ and variance 0.0004. In order to detect a change in mean, 4 samples are taken hourly and tested. If the sample mean falls more than 3 standard errors from the process mean, the process is stopped so that it can be corrected.

i. Determine the 3-sigma control limits for this process **(3 Marks)**

ii. What is the probability that the sample mean will fall out of control limits? **(4 Marks)**

iii. What is the expected number of samples until the chart signals? **(3 Marks)**

iv. Suppose the process mean is shifted to $\mu = 5.11$, what is the expected number of samples until the chart signals **(4 Marks)**

b) A company producing mechanical components uses an acceptance sampling plan with a lot size of 1000, a sample size of 50, and an acceptance number of 2 (i.e., if more than 2 defective items are found, the lot is rejected). The defective rate is 3%.

i. What is the probability that the lot will be rejected (producer's risk)? **(3 Marks)**

ii. If the defective rate is reduced to 1%, what is the new producer's risk? **(3 Marks)**

QUESTION FOUR: (20 MARKS)

a) A process is producing a batch with a specification range of 40 to 60. The process has a mean of 50 and a standard deviation of 3 units. However, the process is shifted, and the actual mean is now 54 units.

i. Compute the C_p and C_{pk} indices. **(5 Marks)**

ii. Comment on the capability of the process after the shift. **(2 Marks)**

b) A factory produces a batch of 1000 products daily, and the number of defective items is recorded for 25 days. The following are the recorded data:

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No. of Defectives	45	50	47	39	53	42	49	46	48	44	41	54	52	48	46
Day	16	17	18	19	20	21	22	23	24	25					
No. of Defectives	43	50	47	49	45	43	51	53	46	44					

i. Construct an *np*-chart for the data **(3 Marks)**

ii. Plot the np-chart and determine if the process is in control **(6 Marks)**

c) An acceptance sampling plan is used where a sample of 80 units is selected from a batch, and the acceptance number is 2. If the true defective rate is 2%, calculate the producer's risk. **(4 Marks)**

QUESTION FIVE: (20 MARKS)

a) In a quality control process, an *np*-chart is being used to monitor the number of defective items in batches of size 500. Over the past 20 samples, the total number of defective items observed is 180. Calculate the center line, UCL, and LCL for the *np*-chart. **(4 Marks)**

b) Suppose that a single sampling plan is to sample 100 items out of a lot of 2000 items and the lot is accepted if the number of defectives is not more than **c**, where **c** is any positive constant. Suppose **c** takes values 0, 1 and that the proportion of defective items takes the values 0, 0.01, 0.02, 0.03, 0.04 and 0.05,

i. Determine the number of defective items and the corresponding probability of accepting a lot. **(5 Marks)**

ii. Obtain the values of np and **P(a)** when **c** is altered such that **c = 1, 2, 3** **(6 Marks)**

iii. Using the standard values of α and β, investigate the effect of the alterations in (ii) above **(5 Marks)**